UDC 351.86
DOI: 10.15587/2312-8372.2019.173640

**Stetsiuk Y.**

# DEVELOPMENT OF MATHEMATICAL MODEL OF LOCALIZATION OF A SMALL EXPLOSIVE OBJECT WITH THE HELP OF A SPECIALIZED PROTECTIVE DEVICE

*В роботі у якості об'єкта дослідження розглядається захисний пристрій куполоподібної форми, який використовується піротехнічними підрозділами для локалізації надзвичайної ситуації у разі вибуху всередині нього малогабаритного небезпечного предмету. Відмічено, що одним з найбільш проблемних місць його застосування є розробка рекомендацій, реалізація яких повинна забезпечити недопущення розвитку надзвичайної події до рівня надзвичайної ситуації за такими пріоритетними наслідками, як кількість жертв та кількість постраждалих. Показано, що визначення таких рекомендацій щодо локалізації наслідків надзвичайної ситуації у разі вибуху малогабаритного вибухонебезпечного предмету за допомогою спеціалізованого захисного пристрою вимагає отримання математичної моделі локалізації осередку надзвичайної ситуації. Ця модель повинна забезпечити оцінку міцності технічного засобу локалізації осколків та стати основою для коригування існуючих оперативних процедур у разі його застосування піротехнічними підрозділами. В ході дослідження використовувався Ейлерово-Лагранжевий підхід, який дозволив отримати математичну модель локалізації за допомогою захисного пристрою куполоподібної форми наслідків надзвичайних ситуацій у разі вибуху всередині нього малогабаритного небезпечного предмету. Математична модель на практиці реалізується в кінцево-елементному пакеті із застосуванням бібліотеки комп'ютерного комплексу ANSYS/AUTODYN. Це дозволяє не створювати кожен раз фактично новий пакет прикладних програм, як це робилось раніше для опису подібних моделей. Завдяки цьому забезпечена оцінка міцності технічного засобу локалізації осколків. У порівнянні з аналогічними відомими моделями розроблена математична модель дозволяє оцінити розмір мінімальної товщини захисного пристрою. Це дозволяє витримати вибух малогабаритного вибухонебезпечного предмету та визначити мінімальну вагу засобу захисту з урахуванням оперативних можливостей бойового розрахунку піротехнічного підрозділу.*

**Ключові слова:** *захисний пристрій, математична модель локалізації вибуху, міцність технічного засобу, пакет ANSYS.*

*Received date: 04.12.2018*
*Accepted date: 18.12.2018*
*Published date: 30.06.2019*



## 1. Introduction

Pyrotechnic units of the civil defense operational and rescue service are engaged in cleaning up the facilities and terrain of Ukraine from explosive remnants of wars that have occurred in the past and are currently taking place. Increasing the operational readiness of pyrotechnics requires an improvement in their technical equipment. One of the directions of this activity is the localization of small-sized explosive objects [1]. However, the lack of appropriate scientific support hinders not only the creation of new types of protective devices, but also their implementation in practice.

Therefore, the study of a perspective dome-shaped protective device, which should provide localization of an emergency situation in the event of an explosion of a small-sized dangerous object inside it, is relevant.

## 2. The object of research and its technological audit

The object of research is a dome-shaped protective device, which is used by pyrotechnic units to localize an emergency in the event of a small dangerous object exploding inside it. Such a device relates to a special technique that is supposed to be used for the safe destruction of small-sized explosive objects of various origins at the place of their detection or transportation to specially equipped areas. The case of the protective device is a cap with the possibility of placing explosive objects under it, handrails for easy carrying and installation, and an increase in weight placed around the perimeter of the base of the case to fit snugly to the surface.

From the standpoint of localizing the consequences of emergency situations (emergency) in case of an explosion of a SEO (small explosive object) and the use of additional load in the practice of pyrotechnic units, the corresponding physical model in the case of the use of a protective device looks like that shown in Fig. 1.

In Fig. 1 there are following definitions:

$A_1$ – small explosive object with a mass $m_{SEO}$;

$C$ – additional load with total mass $m_2$;

$D$ – splinter absorber with a mass $m_3$;

$r_1$ – the longest length from the conventional axis passing to the edge of the splinter absorber $C$;

$E$ – surface on which SEO is located;

$H_{FLY}$ – the height of a flying a technical mean with an additional load as a result of SEO explosion;

$A_2$ – detonation device with a mass $m_{dd}$;

$h_1$ – height of device $B$;

$h_3$ – height of the splinter absorber $C$;

$h_{SEO}$ – SEO height.

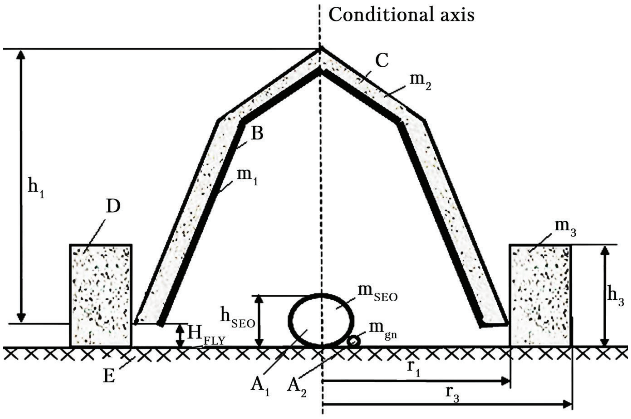


**Fig. 1.** Physical model of localization of the consequences of an explosion of a small-sized explosive object using a protective device

Accordingly, the condition for the effectiveness of a protective device is the strict implementation of the system of equations:

$$\begin{cases} q_1(m,v,t)=0; \\ q_2(m,v,t)=0; \\ q_3(m,v,t)\le q_3^{ob.l}, \end{cases} \quad (1)$$

where $q_1, q_2, q_3$ – respectively, the consequences of emergencies in terms of priority; $m$ – total mass of means necessary to prevent emergencies due to the threat of SEO explosion; $v$ – speed of implementation of measures to prevent emergencies; $t$ – total time for carrying out activities of an operational nature to prevent an emergency; $q_3^{ob.l}$ – quantitative characteristics of indicators of the consequences of emergencies of the third group of priorities, corresponding to the object level.

The ranges of input parameters variation depend on the conditions (Table 1) of both operational and technical nature.

**Table 1**

Impact on the effectiveness of emergency response due to the threat of explosion of a small explosive object

| Type of impact | Parameter | | |
|---|---|---|---|
| | $m$ | $v$ | $t$ |
| Technical conditions | SEO power; strength of protective device B; dimensions of device B; diversity of the behavior of the surface E, which is SEO | Regulatory additional load on pyrotechnics | Expert assessment of the technical condition and danger of SEO |
| Operational conditions | Standard payload weight | State of physical readiness of personnel | The number of personnel of the pyrotechnic division, which is involved |

Analysis of Table 1 shows that preventing emergencies due to the threat of SEO explosion requires the development of recommendations, the implementation of which should prevent the development of an emergency to the emergency level on the following priority consequences, such as the number of victims and the number of victims. Determining such recommendations for localizing the consequences of an emergency in case of SEO explosion with the help of a specialized protective device requires obtaining a mathematical model of localizing the emergency area based on the formation of the characteristics of the corresponding operational and technical measures taking into account a number of the following assumptions:

– the whole mass of operational and technical equipment must be delivered to the cell of a possible explosion:

$$m = m_1 + m_2 + m_3, \quad (2)$$

where $m_1$ and $m_2$ – mass of active agents, which depends on the SEO mass, the mass of the possible remote device, as well as the strength characteristics of the protective equipment and the surface on which SEO is located; $m_3$ – mass of passive elements of the means of localization, which in accordance depend on the height of the vertical movement of active funds and in fact the condition for determining their weight. That is, they can be determined on the basis of the overall parameters of the technical protection means B;

– other technical conditions and operational conditions are the conditions of the limiting type, which allow to obtain the final solution of the model, which is formed as a set of solutions of individual tasks in accordance with each weight component.

## 3. The aim and objectives of research

*The aim of research* is development of a mathematical localization model with the help of a dome-shaped protective device for the effects of emergency situations in the event of an explosion of a small dangerous object inside the protective equipment.

The objectives of research:

1. Justification of assumptions about the conditions of use of the protective device.

2. Determination of restrictions in the application of the selected method of research.

3. Determination of the system of equations describing the localization process using a dome-shaped protective device for the consequences of an emergency in the case of SEO explosion inside the protective equipment, and clarifying the boundary conditions for their use.

## 4. Research of existing solutions of the problem

Determination of recommendations for operational actions related to the localization of explosive objects is considered from two different positions. First, from the point of view of the analysis of existing algorithms for the combat work of the personnel of the pyrotechnic subdivision [2]. Secondly, from the point of view of the explosive destruction of pyrotechnics, when they use personal protective equipment for sappers of different classes [3, 4].

Questions of working with specialized protective devices for collective use are not considered. This is primarily due to the fact that today the greatest difficulties arise [5] when an explosive device is detected and neutralized. At the same time, the use of explosives throughout the world for carrying out terrorist acts revived interest both in the study of explosions and in the study of ways to prevent or mitigate damage from the use of explosives [6]. An empirical way to solve problems in this area [7] confirmed that there are serious problems in this area. Theoretical studies in the field of explosion protection can be divided into two areas. First, it is an understanding of how loads from blast and shock waves propagate. In most cases, their passage through the medium is analyzed; it has a different physical composition [8, 9]. The analysis of such loads when passing through various geometric shapes is also carried out [10, 11]. And, secondly, the study of mitigating mechanisms to minimize damage from impact and explosive load [5].

In [12] it is noted that specific operational recommendations, as a rule, should be based on the results of mathematical modeling of complex explosion scenarios. At the same time, most of the existing mathematical models [13, 14] are based on solving the Euler conservation equation for mass, energy, and momentum. The complex interaction of explosive and shock waves with material configurations and structures requires the use of computers with improved computing power [15], including to predict the effects of an explosion using artificial neural networks [16]. At the same time, in fact, a new software package is created each time to solve the developed mathematical model.

At the same time, today there is a finite-element package «ANSYS» [17], which allows modeling the behavior of a heterogeneous medium under the influence of a pulsed load, based on the Eulerian-Lagrangian approach.

That is, the determination and adjustment of existing operational procedures of pyrotechnics requires the development of a mathematical model of the localization of an explosion of a compact dangerous object inside a dome-shaped protective device. Such a model should provide an assessment of the strength of a technical device for localizing fragments using the ANSYS finite-element package. Justifying the recommendations based on the results of the corresponding modeling will improve the effectiveness of the combat activities of the pyrotechnic units without reducing the level of personnel safety. It will also provide the possibility of repeated use of protective equipment for destruction of explosive objects, and for their transportation.

## 5. Methods of research

To solve the problem of assessing the strength of technical means of localizing fragments, the method of finite element modeling is used, which is implemented in the finite element package ANSYS [17]. Today, this complex allows to simulate the behavior of a heterogeneous medium under the influence of a pulsed load. Inside the protective device are contained both the gaseous medium in the form of detonation products of a small-sized explosive object and air under the surface of the protective equipment, and a solid metal surface. Based on this, development of a mathematical model of localizing the consequences of emergency situations in the case of an explosion of a small dangerous object and the definition of appropriate restrictions is based on the Eulerian-Lagrangian approach.

## 6. Research results

The mathematical model is developed with the assumptions that there is a pulsed load, and SEO is on a complex surface with a penetration factor of 1, and a height $h_{surf}$ that is determined by:

$$h_{surf} = h_1 - h_{SEO}, \quad (3)$$

and corresponds to Fig. 2.

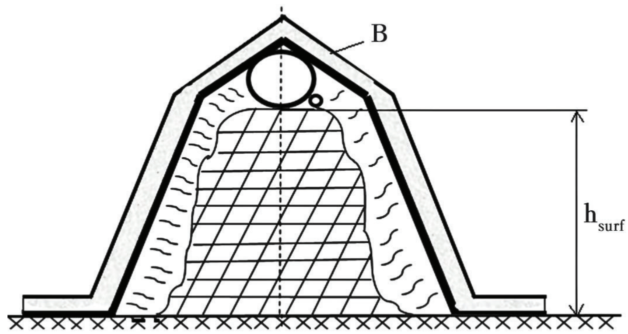


**Fig. 2.** The design scheme for determining the weight characteristics

The chosen SEO model is the worst case in terms of the occurrence of deformations and unprofitable stress in the protection means B. Obtained according to the scheme shown in Fig. 2, the height value of means B will provide the conditions for its strength and for less complex conditions a priori.

Since the simulation of the emergency localization process using a protective device involves the use of the ANSYS end-element package, the expansion of detonation products can be described by the Jones-Wilkins-Lee equation:

$$p = A\cdot\left(1-\frac{\omega}{R_1\cdot V}\right)\cdot e^{-R_1\cdot V} + B\cdot\left(1-\frac{\omega}{R_2\cdot V}\right)\cdot e^{-R_2\cdot V} + \frac{\omega\cdot E_0}{V}, \quad (4)$$

where $p, E_0, V$ – pressure, internal energy and relative volume, respectively; $A, B, R_1, R_2, \omega$ – empirical constants.

This equation is included in the library of the ANSYS/AUTODYNTA computer complex, which solves the problem of determining the structural strength of protective equipment B in an explosion. And thereby determine the weight and size characteristics of the means B, in accordance with which it is used for the intended purpose of the combat calculation of the pyrotechnic unit.

The indicated equation (4) should be supplemented with L. Orlenko isentropic extension equation [18]:

$$p_S = B_0\cdot p^k + C_0\cdot p^{\Gamma}, \quad (5)$$

where $B_0, C_0, k$ – parametric constants of the equation; $G$ – Gruneisen coefficient.

This approach allows to cover the entire range of explosive substances, both industrial and improvised.

The coefficient $G$ value ranges from 0.25–0.35 for small values of density $p$=1000–1600 kg/m$^3$ to 0.7–1 for $p \geq 10000$ kg/m$^3$.

The parameters $B_0, C_0, k$ are determined through the parameters of the detonation wave at the Chapman-Jouget point [18] as follows:

$$B_0 = \frac{p_n - G\cdot\rho_n\cdot E_n}{\rho_n^k}\cdot\frac{k-1}{k-1-G}, \quad (6)$$

$$C_0 = \frac{p_n - B_0\cdot p_n^k}{\rho_n^{1+G}}, \quad (7)$$

$$k = 1 + \frac{2-G}{1-\frac{G}{6}\cdot\left(1+\frac{32Q}{D^2}\right)}, \quad (8)$$

where $Q$ – heat of explosive charge conversion, J/kg; $D$ – velocity of the detonation wave in charge, m/s; $p_n$ – pressure, Pa; $\rho_n$ – density, kg/m$^3$; $E_n$ – energy, J, at the front of the detonation wave.

The corresponding parameters on the front of the detonation wave are determined using the following dependencies:

$$\begin{cases} p_n = \dfrac{\rho_0 \cdot D^2}{4}; \\ \rho_n = \dfrac{4}{3} \cdot \rho_0; \\ E_n = \dfrac{p_n}{2} \cdot \left( \dfrac{1}{\rho_0} - \dfrac{1}{\rho_n} \right) + Q, \end{cases} \qquad (9)$$

where $\rho_0$ – initial density of the explosion.

In the above formulation (4)–(9), the problem of calculating the strength of the body in the specified finite element package is not solved. In this regard, let's introduce the assumption that the case of protective equipment B is made with regard to:

$$h_b = 3 \cdot h_0, \qquad (10)$$

where $h_b$ – body thickness; $h_0$ – the minimum thickness that can withstand SEO explosion, equivalent to 200 g of TNT.

With this in mind, the model for calculating the stress-strain state is a boundary problem of the theory of elasticity, which is completely determined by the system of Navier equilibrium equations:

$$\sigma_{ij,j} + F_i = 0, \; i, j = 1,2,3, \qquad (11)$$

where $\sigma_{ij,j}$ – the components of the stress tensor; $F_i$ – the components of the vector of bulk forces, which are determined on the basis of the solution of equation (4) for known industrial designs of an explosive substance or from the solution of the system of equations (5)–(9) for explosive substances of nonindustrial manufacture.

The Cauchy formulas, which establish the relationship between the components of the displacement vector $U$ and the strain tensor $E$, have the form:

$$E_{ij} = \frac{1}{2} \cdot \left( U_{i,j} + U_{j,i} \right), \; i, j = 1,2,3, \qquad (12)$$

and physical relationships:

$$\sigma_{ij} = \frac{\partial W\left(E_{ij}\right)}{\partial E_{ij}}, \; i, j = 1,2,3, \qquad (13)$$

are supplemented by boundary conditions:

$$p_{iv} = \sigma_{ij,n_j}, \; E_i \in S_F, \qquad (14)$$

$$U_i = U_{i0}, \; E_i \in S_U, \qquad (15)$$

where $W\left(E_{ij}\right)$ – energy of elastic deformation; $n_j$ – direction cosines of the external normal to the boundary of the region; $S_U$ and $S_F$ – sections of the boundary where the given boundary conditions in displacements and loads, respectively; $p_{iv}$ – components of the vector of surface forces.

Accordingly, the task in the variational formulation is from the condition of minimum variation of the total energy:

$$\partial E\left(U\right) = 0, \qquad (16)$$

can be represented as:

$$E\left(U\right) = \iiint\limits_V W\left(E_{ij}\right) \mathrm{d}V - \iiint\limits_V F_i U_i \mathrm{d}V - \iint\limits_{S_F} P_{iU} U_i \mathrm{d}S, \qquad (17)$$

where $E\left(U\right)$ – the total energy of the entire elastic region.

Given the Kostelyanko dependencies relative to the specific potential energy:

$$\frac{\partial W(E_{ij})}{\partial \sigma_{ij}} = E_{ij}, \qquad (18)$$

equation (17) takes the form:

$$E\left(U\right) = \frac{1}{2} \iiint\limits_V E^T \sigma \mathrm{d}V - \left( \iint\limits_{S_F} U^T P \mathrm{d}S + \iiint\limits_V U^T F \mathrm{d}V \right), \qquad (19)$$

where $T$ – the transpose operation.

Consequently, a mathematical model of localization with the help of a protective device of the consequences of emergency situations in the case of an explosion of a small-sized dangerous object is a system of equations (4), (5), (9) and (19) with due regard for requirements (10), (14) and (15). The presence of such a mathematical model allows for calculations of the strength of the instantaneous load as a result of SEO undermining, relying on the libraries of the ANSYS/AUTODYN computer complex when modeling.

That is, the solution of the developed mathematical model allows to estimate the size of the minimum thickness $h_0$, which will withstand SEO explosion equivalent to 200 g of TNT, and taking into account the material of protective equipment B, namely steel 20 with parameters $p$=7850 kg/m$^3$; $E$=210 GPa; $\mu$=0.33; $\sigma_T$=288 MPa; $E_t$=1 GPa. Accordingly, it is possible to determine the minimum weight of protective equipment B, taking into account the operational capabilities of the combat crew of the pyrotechnic unit.

## 7. SWOT analysis of research results

*Strengths.* The ability to calculate on the durable means of protection of personnel of pyrotechnic units and the population as a result of the explosion of a small explosive object inside a dome-shaped protective device, based on the results of modeling using the existing library computer complex ANSYS/AUTODYN. The calculation results make it possible to minimize the weight of this type of protective equipment and, consequently, to increase the operational capabilities of the pyrotechnic unit in the process of localizing emergency situations associated with explosive objects.

*Weaknesses.* The complexity of the preparation of the initial data requires for the implementation of this process highly qualified specialists. Such specialists must simultaneously have knowledge both in the theory of detonation and strength, and in the practice of conducting work on the controlled destruction of explosive objects.

ISSN 2226-3780

*Opportunities*. Improve operational procedures for the elimination of small-sized explosive items by establishing the ratio between the TNT equivalent of a small-sized explosive item, the weight and size characteristics of the protective equipment and the mass of the additional load. Expand the range of multifunctional protective devices, in the case of determining the positions of their use for the temporary isolation of explosive items on objects with a massive stay of people, as well as in the transport of explosive items to the place of elimination.

*Threats*. An incorrect (understated) expert assessment of the properties of an explosive object can lead not only to the destruction of the protective equipment, but also to human casualties.

## 8. Conclusions

1. The rationale for assumptions for the development of a mathematical model is done. The worst in terms of the occurrence of deformations and unprofitable stress in the means of protection conditions of its use are selected. First, it is a pulse type of load. Secondly, such a scheme of placement of a small explosive object, when it is on a complex surface with a penetration coefficient of 1, and touches the upper part of the dome of the protective device. Obtained according to this scheme, the high value of the means of protection will ensure its strength and under less difficult conditions a priori.

2. The definition of restrictions in the application of the selected research method is made. The task of determining the strength of a dome-shaped protective device should be solved using the library computer complex ANSYS/AUTODYN.

3. A definition of the system of equations (4), (5), (9) and (19) is made, which makes it possible to obtain a mathematical model for localizing the consequences of an emergency in the event of SEO explosion inside a dome-shaped protective device. The solution of this model, taking into account the boundary conditions (10), (14) and (15), makes it possible to estimate the size of the minimum thickness of the protective device and determine its minimum weight, taking into account the operational capabilities of the personnel.

### References

1. *Pro skhvalennia Stratehii reformuvannia systemy DSNS Ukrainy* (2017). Rozporiadzhennia Kabinetu Ministriv Ukrainy No. 61. 25.01.2017. Baza danykh «Zakonodavstvo Ukrainy». VR Ukrainy. Available at: https://zakon.rada.gov.ua/laws/show/61-2017-%D1 %80
2. Xiao, T., Horberry, T., Cliff, D. (2015). Analysing mine emergency management needs: a cognitive work analysis approach. *International Journal of Emergency Management, 11 (3)*, 191–208. doi: http://doi.org/10.1504/ijem.2015.071705
3. Toan, D. Q. (2015). Train-the-Trainer Trauma Care Programin Vietnam. *Journal of Conventional Weapons Destruction, 19 (1)*. Available at: http://commons.lib.jmu.edu/cisr-journal/vol19/iss1/9
4. Smith, A. (2017). An APT Demining Machine. *Journal of Conventional Weapons Destruction, 21 (2)*. Available at: http://commons.lib.jmu.edu/cisr-journal/vol21/iss2/15
5. Hadjadj, A., Sadot, O. (2013). Shock and blast waves mitigation. *Shock Waves, 23 (1)*, 1–4. doi: http://doi.org/10.1007/s00193-012-0429-0
6. Tyas, A., Rigby, S. E., Clarke, S. D. (2016). Preface to special edition on blast load characterisation. *International Journal of Protective Structures, 7 (3)*, 303–304. doi: http://doi.org/10.1177/2041419616666340
7. Blakeman, S. T., Gibbs, A. R., Jeyasingham, J. (2008). A study of mine resistant ambush protected (MRAP) vehicle as a model for rapid defence acquisitions. *MBA Professional Report Monterey Naval School*. Available at: http://www.dtic.mil/dtic/tr/fulltext/u2/a493891.pdf
8. Sherkar, P., Whittaker, A. S., Aref, A. J. (2010). Modeling the effects of detonations of high explosives to inform blast-resistant design. *Technical Report MCEER-10–0009*. Available at: https://ubir.buffalo.edu/xmlui/bitstream/handle/10477/25356/10-0009.pdf?sequence=3
9. *Armor Thane Reduces the Impact from Bombs and Bullets*. Available at: https://www.armorthane.com/protective-coating-applications/blast-mitigation-protection.htm
10. Togashi, F., Baum, J. D., Mestreau, E., Löhner, R., Sunshine, D. (2010). Numerical simulation of long-duration blast wave evolution in confined facilities. *Shock Waves, 20 (5)*, 409–424. doi: http://doi.org/10.1007/s00193-010-0278-7
11. Snyman, I. M., Mostert, F. J., Olivier, M. (2013). Measuring pressure in a confined space. *27th international symposium on ballistics, 1*, 829–837.
12. Woodley, C., Feng, C., Li, B. (2018). Defence Technology. 1st *International Conference on Defence Technology*. Beijing, 14 (5), 357–642. doi: http://doi.org/10.1016/s2214-9147(18)30442-2
13. Van den Berg, A. C. (2009). «BLAST»: A compilation of codes for the numerical simulation of the gas dynamics of explosions. *Journal of Loss Prevention in the Process Industries, 22 (3)*, 271–278. doi: http://doi.org/10.1016/j.jlp.2008.07.004
14. Cullis, I. G., Nikiforakis, N., Frankl, P., Blakely, P., Bennett, P., Greenwood, P. (2016). Simulating geometrically complex blast scenarios. *Defence Technology, 12 (2)*, 134–146. doi: http://doi.org/10.1016/j.dt.2016.01.005
15. Chaudhuri, A., Hadjadj, A., Sadot, O., Ben-Dor, G. (2012). Numerical study of shock-wave mitigation through matrices of solid obstacles. *Shock Waves, 23 (1)*, 91–101. doi: http://doi.org/10.1007/s00193-012-0362-2
16. Remennikov, A. M., Mendis, P. A. (2006). Prediction of airblast in complex environments using artificial neural networks. *WIT transactions on the build environment, structures under shock and impact IX*, 269. doi: http://doi.org/10.2495/su060271
17. *Programmnyi paket ANSYS*. Available at: https://sites.google.com/site/komputernoemodelirovanie/home/stati/programmnyj-paket-ansys
18. Andreev, S. G., Babkin, Iu. A., Baum, F. A. et. al.; Orlenko, L. P. (Ed.) (2002). *Fizika vzryva*. Vol. 1. Moscow: FIZMATLIT, 832.

***Stetsiuk Yevgen***, *Department of Pyrotechnic and Special Training, National University of Civil Defence of Ukraine, Kharkiv, Ukraine, e-mail: stecyuke@gmail.com, ORCID: http://orcid.org/0000-0002-5204-9194*