Запропоновано кореляційний метод оперативного виявлення рекурентних станів в складних динамічних системах при нерегулярних вимірах. Узагальненні поняття кореляції для випадку векторів станів траєкторії динаміки складних систем і оцінки кореляції векторів для рухомого по траєкторії фрагмента фіксованої протяжності. Для реалізації методу використовується простір зі скалярним добутком векторів станів. Оцінка величин кореляцій векторів станів дозволяє інтерпретувати їх в якості відповідних рівнів енергетичної взаємодії векторів станів і здійснювати виявлення ступеня їх рекурентності. При цьому обчислення величини кореляцій здійснюється тільки на основі відомих вимірювань вектора станів і не вимагає визначення порогу і способу обчислення відстані, традиційно використовуваних в методах рекурентних діаграм. Працездатність запропонованого методу проводилася на конкретному прикладі експериментальних даних реальної динаміки вектора станів забруднень міської атмосфери. У якості складових вектора станів розглядалися газові забруднювачі: формальдегід, аміак і оксид вуглецю. Отримані результати в цілому свідчать про працездатність запропонованого методу. Експериментально встановлено, що кореляційний метод в разі нерегулярних вимірювань атмосферних забруднень забезпечує автентичність виявлення рекурентних станів, відповідних максимальної кореляції станів. В цьому випадку оцінка кореляцій повинна проводитися для рухомого фрагмента траєкторії вектора станів. При цьому протяжність фрагмента повинна складати не більше 10 відліків

Ключові слова: кореляція станів, енергетична взаємодія, фрагмент траєкторії станів, рекурентні стани, складні динамічні системи, газові забруднення атмосфери

DEVELOPMENT OF THE CORRELATION METHOD FOR OPERATIVE DETECTION OF RECURRENT STATES

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1. Introduction

Most actual objects of natural and artificial origin are complex non-linear dynamic systems. The analysis of current states, as well as the forecast of dynamics of states for such systems, is an important scientific and practical problem that has not yet been completely solved. It was established that the dynamics of states of these systems is not arbitrary. It follows the well-known fundamental principle of dissipative dynamic systems, which implies the recurrence of their

states [1]. A constructive tool for studying the recurrence of states of complex dynamic systems is the methods of recurrent plots (RP) [2]. These methods belong to the visual class and are the basis of modern methods for recurrent quantitative assessment (RQA) of states. The RP and RQA methods are widely used in a variety of applications [1, 3–6]. However, the capabilities of the well-known RQA methods depend significantly on the authenticity of displaying recurrent states using the visual methods of the RP. Objective complexity and uncertainty of dynamic systems, as well as the need for authentic display of the recurrent states of systems create the problem of improving the known methods for detection of recurrent states in complex dynamic systems.

2. Literature review and problem statement

In discrete measurements, the multidimensional vector of the state of systems in space will be displayed by the corresponding points moving discretely along the current trajectories of the system states. The states recurrence at discrete moments of time will mean that the correspondent trajectory points appear to be close in a sense [4]. For example, the methods for detection of correlation dimensionality [5] used to detect recurrent states, unlike the RP methods, do not make it possible to display visually the multidimensional states of systems into a plane (2-dimensional space). It is known that it is possible to restore multidimensional states of complex dynamic systems with their subsequent displaying in the form of RP by measuring only one of the coordinates of a multidimensional vector of states [6]. For example, prediction of hazardous conditions of polluted atmosphere of industrial cities based on the RP calculation by the measurements of one of the coordinates of the vector of pollution states of pollution is considered in paper [7]. The calculation of the RP is limited to the usual metric (distance) in space of all actual numbers. Paper [8] describes the use of the RP method for a 5-dimensional vector of states of wind velocities in five districts of Nigeria. The results are limited to the consideration of linear space and Euclidean metric. Other metrics and spaces, as well as irregularity of measuring the wind velocity vector, are not considered. The study is devoted to the problem of removing artifacts during the calculation of the RP. However, the study is limited to the consideration of the Euclidean metric of phase space. The methods for calculating the RP of the reconstructed vector of states the Earth's magnetosphere are considered in article [10]. At the same time, the methods for the RP calculation are limited to normalized linear spaces with the maximum metric and the Chebyshev metric. Paper [11] examines the application of the RP methods for recognition and classification of human motor activity. It is noted that the application of the RP methods in this case has low authentication, mainly due to threshold uncertainty of the methods. To ensure the authentication of motor activity, it is proposed to perform recognition and classification directly by the distance matrix, which does not depend on the threshold with the subsequent use of neural networks. However, the use of neural networks is related to the known shortcomings and limitations. The application of the RP methods to studying the behavior of biosystems is considered in article [12]. The RP is calculated in a space with the Euclidean metric. At the same time, one of the important applications of these methods is considered to be the correspondence (authenticity) of the calculated RP to recurrent states of actual studied systems. This property will be subsequently considered as a property of authenticity of the RP calculation methods. The authenticity of the RP methods is significantly influenced by the nature of the measurement conditions, the operator of the norm calculation, the magnitude of the time delay and dimensionality of the attachment, as well as the magnitude of the recurrent threshold. Paper [13] deals with the calculation of the RP at irregular measurements. In this case, calculations are limited to considering space with a normal metric of distance. Article [14] considers general recommendations to overcome the threshold uncertainty of the known RP methods, which involves fixing the threshold, depending on a specific task of the study. Paper [15] explores the identification of the features in the dynamics of states of actual dynamic systems based on the RP calculation. It is argued that the threshold magnitude should be some function of standard measurement deviation. In this case, the type of function is not specified, but it is only noted that this method for determining the threshold magnitude can be useful for any of the RP calculation methods. The combination of a multi-level network approach and recurrent networks to analyze state dynamics in multi-dimensional phase spaces is considered in paper [16]. At the same time, the studies are limited to considering the multidimensional vector of states only in normalized space with the Euclidean metric. Other types of metrics, spaces, and methods for assessing recurrent states are not considered. Article [17] focuses on the specifics of the modern RP and RQA methods and their applications. It is noted in it that the RP and RQA methods can use spaces of different dimensionality with different types of metrics. However, the issues of the impact of a metric and the threshold magnitude on the authenticity of the RP display are not addressed. Possible methods for identifying recurrent states in complex dynamic systems based on the principles other than the RP, such as correlation, are not discussed. Paper [18] examines the specifics of the use of the RP of carbon monoxide concentrations for early detection of ignitions in non-pressurized premises. The research results are found for one-dimensional space with normal and power metrics of distances. The peculiarities of the RP calculation at irregular measurements of states are not discussed. The studies in multidimensional spaces and the possibilities of the threshold adaptation are not considered. At the same time, the methods and devices of the threshold adaptation at the detection of early ignitions are explored in [19]. It is noted that the threshold adaptation is crucial in detecting early ignitions [20]. At the same time, correlation principles of detection are not considered.

Thus, the detection of recurrent states in complex dynamic systems is primarily based on the calculation of the RP and the use of normalized spaces with different metrics. It should be considered that the most studied methods are the methods for RP calculation using linear spaces with uniform, Euclidean and maximum metric. In this case, the RP, calculated for the same system in spaces with different metrics, will be different. The implementation of the RP methods is based on the results of all measurements of the system's states on the given observation interval. That is why the detection of recurrent states in real time based on the use of the RP is problematic. The methods for detection of recurrent states, based on other principles in normalized spaces with different types of metrics, should be considered not sufficiently explored. The authenticity of the display of recurrent states

in the RP depends significantly on the chosen measure and the threshold magnitude and other parameters. At the same time, the well-known heuristics [14] of threshold selection are particular, usually associated with the used space metrics and having significant implementation limitations. In this regard, an important and unresolved part of the problem of the improvement of the methods for detection of recurrent states in complex dynamic systems is the development of the methods for operative detection of authentic recurrent states based on the use of the correlation approach.

3. The aim and objectives of the study

The aim of the present research is to develop the correlation method for operative and authentic detection of recurrent states in complex dynamic systems of different nature at irregular measurements of the vector of states.

To achieve the aim, the following tasks were set:

– to substantiate the correlation method for operative and authentic detection of recurrent states in complex dynamic systems at irregular measurements of the vector of states;

– to test experimentally the effectiveness of the proposed correlation method for operative detection of recurrent states in complex dynamic systems on the example of an actual system of pollution of urban air with typical objects of critical infrastructure.

4. Substantiation of the correlation method for operative and authentic detection of recurrent states in complex dynamic systems

The detection of recurrent states in complex dynamic systems is usually based on processing actual measurements of current states of the system. In practice, measurements are most often taken at discrete moments of time i and on the assigned observation interval. In this case, the results of measurements at discrete moments of time i are presented in the form of a sequence of m-dimensional vectors Z_i.

The RP methods make it possible to display the measurable states of actual complex dynamic systems in the form of the corresponding matrix of states $R_{i,j}^{m,\varepsilon}$. Following [21], the totality of the known RP methods can be represented in a generalized form:

$$R_{i,j}^{m,\varepsilon} = \Theta\varepsilon\left(-\Delta_{i,j}\right), \quad \Delta_{i,j} = \left\|Z_i - Z_j\right\|,$$

$$Z_i \in \Omega^m, \quad Z_j \in \Omega^m, \quad i,j = 0,1,2,...,N_S - 1, \qquad (1)$$

where $\Theta(\varepsilon - \Delta_{i,j})$ is the Heaviside function; ε is the assigned threshold of recurrence of states; $\Delta_{i,j}$ is the assigned norm of the difference of vectors of states, measured at time moments i and j in m-dimensional space; N_S is the total number of m-dimensional vectors Z_i of states, and Ω^m is the set of all measured vectors of states. Recurrence threshold ε is (1) is determined by a priori assigned radius of the m-dimensional sphere, within the boundary of which vectors Z_i and Z_j for time moment i are recognized as recurrent. We will assume that in the m-dimensional space, each vector of states determines the corresponding point. This means that for the assigned observation interval in m-dimensional space, $N_S - 1$ point will be determined from set Ω^m.

Following (1), the recurrence of states in complex dynamic systems is determined by calculating the distance between the pairs of points corresponding to the assigned pair of vectors of states in m-dimensional space. At the same time, if this distance is less than assigned threshold ε, the decision on the recurrence of this pair of states is made. Detection of the recurrence of states based by the feature of distance is intuitively clear, but not authentic enough. Because it is in some cases quite rough and ambiguous. Roughness is associated with the feature, which takes into consideration only reciprocal remoteness of the points in m-dimensional space without taking into consideration their position in space. The ambiguity is manifested in the relationship between the decision on the recurrence of states on the method for calculating the distance between the corresponding points and the threshold magnitude. At the same time, the choice of the threshold and the method for calculation of the distance are arbitrary. This leads to ambiguity, roughness and the lack of authenticity in the detection of recurrent states for actual systems. It should be noted that the known RP methods are used only when the entire totality of measurements on the assigned observation interval is known. This makes it impossible to apply these methods under conditions when measurements are carried out sequentially at real points of time. That is why in this sense, the known RP methods cannot be considered operative. To ensure promptness, a certain modification of the known RP methods is proposed [7]. However, the proposed modification does not affect the change in the RP principle (1). This means that the modification is aimed only at ensuring promptness of the known methods for RP computation and does not deal with the elimination of other shortcomings noted above (1). To eliminate these shortcomings, it is necessary to change the principle of calculation of recurrent states (1), based on measuring the distances between the points determined by corresponding vectors of states in the corresponding m-dimensional space and comparing them with the established threshold.

It is important to note that m-dimensional vectors Z_i of states, measured at corresponding discrete moments of time on the assigned observation interval, are generally temporary implementations of the non-stationary discrete m-dimensional vector of a random process. This means that in the case under consideration, the trajectory of states is represented as the corresponding set of points in m-dimensional space. Each point is determined by the corresponding vector of states. Recurrence or similarity of the specified vectors of states or trajectory points can be assessed by correlation functions or correlation coefficients that reflect the strength of the relationship between arbitrary variables. The Cheddock scale is usually used to classify the strength of relationship. In general, to have an idea of the similarity between the values of the process, the average quadratic deviation between them over a certain time interval, attributed to interval magnitude, is calculated. Normalization relative to the interval magnitude in this case is only a scale factor, allowing moving from energy to average power. That is why it is more appropriate to estimate the recurrence or similarity of the considered vectors Z_i of states by means of correlation functions related not to power, but to energy of interaction of states. The energy of interaction of states considered at different points in time is a more well-grounded concept to reflect the recurrence and degree of similarity of states, as well as to predict the dynamics of states in the future. This is explained by deep relation of the interaction energy

with the correlation of states in different physical systems. It is known that without energy transfer, it is impossible to exchange information. At the same time, the amount of transferred energy may be small, but this amount in actual physical systems is always different from zero. In this regard, the energy of interaction of the trajectory points should be implied by energy interaction of the points of trajectory of states of the studied system.

The proposed correlation approach, involving the interpretation of energy interaction of the points of the trajectory of states of the studied system, implies that moments of time j, considered with respect to moment of time i in (1), should be apart for time τ. At the same time, each moment j in (1) is related to the corresponding moments $i+\tau$ or $i-\tau$.

Typically, correlation analysis of non-stationary random processes is based on multi-dimensional joint densities of probability distribution or averaging by the set of implementations. However, it is not possible to obtain the set of implementations of m-dimensional vectors Z_i of states for each moment of time i in complex systems. Joint densities of probability distribution are also unknown. In practice, it is possible to obtain only one implementation of such process, but for a fairly long period of time. That is why it does not seem possible to use averaging for many implementations. The only possible variant is the calculation of the temporal correlation function on the limited interval of time is the assumption that a random process is ergodic. This means that the current average operator is used in the general case for correlation analysis of non-stationary processes. The assessment of correlation function is shifted. In the general case, the shift error is directly proportional to square of current average interval. Error dispersion depends on the type of non-stationary process and in the case of Gaussian processes is inversely proportional to square of interval averaging.

It should be clarified that hereafter the term correlation function refers to the definition given in paper [22]. At the same time, correlation function for the considered non-stationary discrete m-dimensional vector random process contains all the information that is known about this trajectory of states of the studied system.

Implementation of the correlation approach is related to the use of spaces, for which the operation of scalar product of vectors is true [23]. Such product is a display of orderly pairs of linear space vectors on the actual axis. This display for an arbitrary pair of system state vectors is convenient to designate through $(Z_i, Z_j) = Z_i^T Z_j$ where T is a sign of transposing. At the same time, scalar product of vectors will generate the norm. For arbitrary vector of state Z_i, the norm will be determined by magnitude $\|Z_i\| = (Z_i, Z_i)^{0,5} = (Z_i^T Z_i)^{0,5}$. Taking into consideration the above designations, the estimation of correlation function for an arbitrary fragment of the trajectory by the measured vectors of states of the studied system will be determined from expression

$$KF_1(N0, NN, \tau) = \frac{1}{NN-\tau} \sum_{i=0}^{NN-\tau-1} Z_{i+N0}^T Z_{i+N0+\tau}, \qquad (2)$$

where $N0$ is the discrete value of the beginning of the trajectory fragment; NN is the number of discrete values determining the trajectory fragment length; τ is the number of discrete shifts (delay) of measurement in definition (1), where $\tau < NN$.

Evaluation of the normalized correlation function (2) in this case will be determined by the expression in the form

$$KFnor_1(N0, NN, \tau) = \frac{KF_1(N0, NN, \tau)}{KF_1(N0, NN, 0)}. \qquad (3)$$

Correlation analysis serves as a means for determining the dominant correlations and lags (delays), as well as hidden periodicities in the process. In addition, in the case under consideration, high correlations of the points of the trajectory of states serve as an indicator of the significant energy interaction of these points, and the lag magnitude indicates a temporary delay in the transmission of the corresponding energy interactions along the trajectory of states. Functions (2) and (3) in this case differ from the known functions by the fact that they make it possible to assess the correlations (energy interaction) of discrete points of the fragment of the trajectory of states, taking into consideration the distances of these points from the beginning of space coordinates and the angle between vectors of states corresponding to these points. In this case, the considered functions (2) and (3) depend on three parameters; besides the traditional parameter of the function delay, they depend on the beginning and the length of the fragment of the trajectory of the system (analysis epoch). At a larger length of the fragment and significant non-stationary nature of the trajectory of states, estimates (2) and (3) will be calculated for fragment sections with different statistical properties. That is why the known interpretation of results based on Pearson's correlation factors turns out to be untrue. In this case, it is more convenient to use the interpretation based on energy interactions of states.

To determine the energy interaction of the points of the trajectory of states, it is possible to use estimates (2) and (3) as function of parameter $N0$ of current discrete beginning of the trajectory fragment. At the same time, the number of discrete values NN that determine the length of the trajectory fragment (analysis epoch) and the delay magnitude τ are fixed. In this case, based on assessment of current energy interaction of trajectory points, it is possible to assess the degree of recurrence of corresponding states, as well as to predict the dynamics of the system states to identify possible hazardous conditions.

The developed correlation approach based on assessment of the current energy interaction of states makes it possible to identify the degree of recurrence of states in complex dynamic systems only if there are measurements of all points of the trajectory of states for an assigned fragment. This means that the assessment of energy interaction of states based on (2) and (3) is not operative and can be used to analyze, study behavior, and detect recurrent states of complex dynamic systems only in the case of all measurements of state for the assigned trajectory fragment. Operative assessment of the energy interaction of discrete points of the trajectory in phase space requires the method that would be based only on measurements prior to the current assessment point and would make it possible to predict the development of the dynamics of the trajectory of states of the system in the future. Given this, the correlation method for operative detection of recurrent states in complex dynamic systems, based on correlation estimates related to energy interactions of states, can be represented by a modification of estimate (2) in the form

$$KF_2(t, \tau, NN) =$$
$$= if\left\{t < NN, 0, \frac{1}{NN-\tau} \sum_{i=0}^{NN-\tau-1} Z_{t-i}^T Z_{t-i-\tau}\right\}, \text{at } \tau < NN. \qquad (4)$$

The modified estimate (4) determines the correlation method for operative detection of recurrence of states in complex dynamic systems based on correlations, interpreted as energy interactions of states. Modification (4) is based on estimate (2), but is made for the trajectory fragment of *NN* length, moving along the entire trajectory of the system states. In this case, at the initial stage, when current time *t=NN*, assessment of correlations of the points of the trajectory fragment (energy interaction of the points) of states is not performed. At this stage, measurements of the states of the assigned initial fragment of the trajectory are only measured and memorized, and estimate (4) is accepted as equal to zero. At the subsequent moments of time, based on the already accumulated data and current measurements, current correlations of the trajectory points are assessed and energy interaction of the corresponding points is estimated on this basis. It should be noted that the length of the moving fragment of the trajectory will have an impact on the initial delay of assessment (4), the shift error, as well as the accuracy of localization in time of correlation and energy interaction of states that determine the degree of recurrence of the states of the system. Estimate (4) is a function of three parameters – current time, lag (delay) and the length of the trajectory fragment. That is why estimate (4) can generally be considered as functions of one, two or three variable parameters. Selection of a specific number of variables and the values of their parameters in (4) is determined by the target of the solved application task. The proposed method makes it possible to obtain results that do not contradict the results obtained based of the RP methods (1) in the case of the choice of the threshold and the norm from the condition of ensuring the authenticity of recurrent state. The results are also comparable to those of the methods of authentic frequency-temporal representation [24].

At the same time, the proposed method does not require the choice of the threshold and the norm for assessing the recurrence of states. Recurrence is assessed by the correlation level and, accordingly, the energy interaction of states. The method is operative because it is based only on the results of actual measurements of the vector states of the studied system proceeding the current moment of assessment. The use of the correlation of states and the fragment moving along the trajectory of states in real time makes it possible to interpret physically the correlation as an appropriate level of energy interaction of states. This makes it possible to ensure the authenticity of the detection of the degree of recurrence of states in complex dynamic systems of different nature without using the recurrence threshold.

5. Experimental test of efficiency of the correlation method for operative detection of recurrent states

The efficiency of the correlation method of operative detection of recurrent states was tested on the example of the actual system of pollution of urban atmosphere by dangerous sources. The most dangerous gas pollutants of the atmosphere, caused by motor vehicles [25], fires [26, 27] and accidents at sites of critical infrastructure, were considered as pollutants [28]. The data on gas pollution of the atmosphere were obtained during irregular measurements of pollution exceeding the maximum allowable concentrations (MAC). When choosing specific gas pollutants, the relations of atmospheric pollution and the greenhouse ef-

fect, acid rains [29] and poisoning of water layers [30] were taken into consideration. That is why exceeding the MAC by formaldehyde (CH_2O), ammonia (NH_3) and carbon monoxide (CO) were chosen as measurable components of the vector of states of atmospheric pollution. The method of experimental measurements and the characteristics of the used equipment are shown in [7]. The test interval of the measured excesses of the MAC by the specified gas pollutants of the atmosphere was determined from 13:00 on May 3, 2018 (*i*=490) till 01:00 of May 11, 2018 (*i*=520). In the course of the experiment, the measurements started from 13:00 on January 1, 2018 (*i*=0). The choice of the specified test interval is related to considerable exceeding the MAC by the selected atmospheric pollutants, which became the sources of a hazardous situation [31].

As an illustration, Fig. 1 shows the normalized correlation functions (3) (red curves) on the test interval as functions of discrete delay (after 6 hours) for two different in length segments of given measurements, but having the same beginning (490 countdown). The measured values (in blue) of exceeding the MAC by formaldehyde in the atmosphere are also shown. The area of exceeding the MAC, which was the source of a hazardous situation, is separated by a rectangle. A dangerous discharge of formaldehyde into the atmosphere occurred between 506 and 507 counts. Similar dependences of the normalized correlation functions (3) were also obtained for other fragments of the measured trajectory of states of atmospheric pollution. The nature of dependences turned out to be similar. At an increase in the duration of the analyzed fragment of the trajectory, the estimates of normalized correlation functions for equal intervals of delay turned out to be smoother.
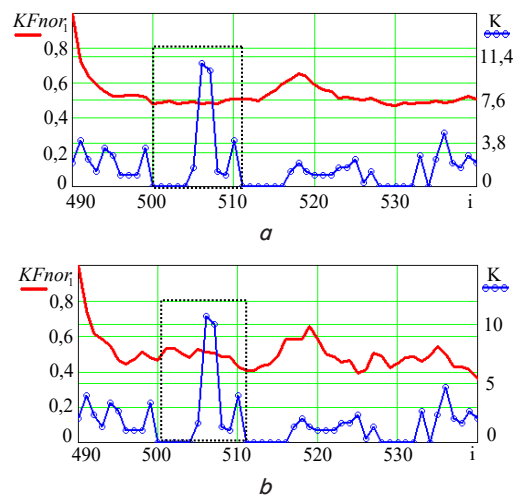

Fig. 1. Normalized correlation functions (3) for the test interval and different length of the fragment of trajectory of the vector of states of pollution: *a* — 600 discrete measurements; *b* — 100 discrete measurements

At the same time, normalized correlation functions do not make it possible to identify the features of pollution dynamics. That is why to identify the specifics of the dynamics of states, the trajectory fragments must be short. At the same time, the delay interval should be even shorter in order to ensure that there is no shift of estimates. In this regard, normalized correlation functions, such as the functions of two parameters – discrete measurement time and discrete delay, were studied. Fig. 2 shows the corresponding cross-sections

of the dispersion of the MAC and the normalized correlation function (3) in the time – delay plane on the interval of the detected dangerous discharge of formaldehyde, separated in Fig. 1 by a rectangle.
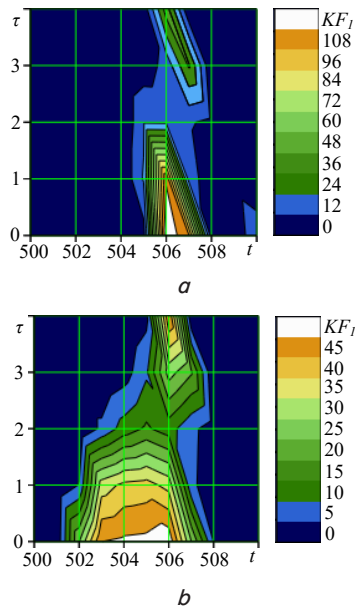

Fig. 2. Cross-sections for dispersion of the vector of states and the normalized correlation function (3):
a — dispersion; *b* — normalized correlation function

The main limitation of the considered correlation methods (2) and (3) is that the methods are efficient only if all measurements are available on the studied interval of time. That is why methods (2) and (3) are not operative. In this regard, the correlation operative method (4) based on (2) was proposed. Fig. 3 shows the corresponding cross-sections of correlation function (4) in the time – delay play on the test interval of atmospheric pollution for two short fragments of the trajectory of different lengths.
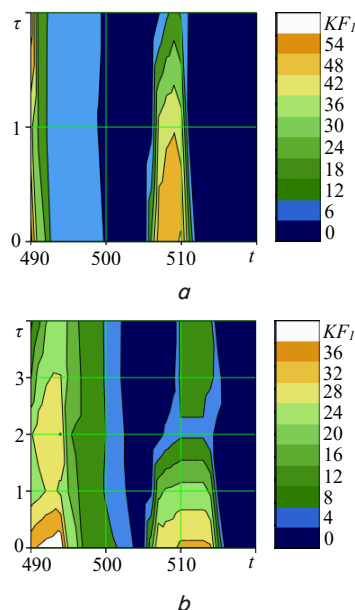

Fig. 3. Cross-sections of correlation function (4) on the test interval for different length of the trajectory fragments:
a — 4 counts (1 day); *b* — 8 counts (2 days)

The results of the influence of length M of the trajectory fragment on the correlation function (4) at various delays are shown in Fig. 4.
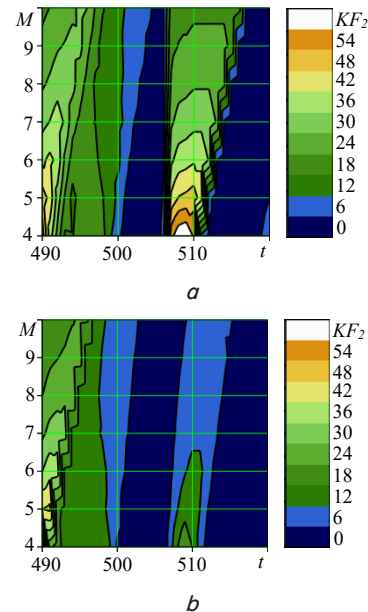

Fig. 4. Cross-sections of correlation function (4) on the hazardous interval of pollution at different delays:
a — 1 count; *b* — 2 counts

Cross-sections of correlation functions (4) at a hazardous interval of pollution shown in Fig. 3, 4 correspond to the proposed correlation method for operative detection of the recurrence of vector states of arbitrary dimensionality in complex dynamic systems. These results generally make it possible to assess quantitatively the level of energy interaction between the corresponding vectors of states of arbitrary dimensionality in complex dynamic systems.

6. Discussion of results from experimental testing of the proposed method

Analysis of the results presented in Fig. 1 reveals that correlation methods (2) and (3) are not operational methods for detecting recurrent states in complex dynamic systems, in a particular case, of various natural systems of measurements of atmospheric pollution. These methods require a priori information about all measurements for the studied interval of analysis. At the same time, methods (2) and (3) make it possible to assess quantitatively the spread of energy interaction of vector states in systems over time relative to the moment of occurrence of disturbances. The cited results indicate that the considered system of atmospheric pollution is not random, but rather chaotic and non-stationary. In addition, the interval of stationarity is quite small (no more than 10 counts). An increase in this interval leads to a decrease in authenticity of identification of recurrent states over time (Fig. 2, *b*). In this regard, the considered intervals of analysis equal to 600 and 100 counts are too rough to detect the non-stationary features of the considered system of atmospheric pollution. The research results, shown in Fig. 2, indicate a significant non-uniformity of distribution of energy of interaction between the vectors of state of atmospheric pollution in the time-delay plane. At the same time,

the maximum value of energy of interaction corresponds to zero delay between 504 and 506 counts. At an increase in the length of the trajectory fragment, the area of distribution of significant values of energy of interactions expands. The area of significant energies of interaction indicates that in these states, there is a reciprocal exchange of energy, and states can be considered recurrent in the sense of energy interaction. In the area of the absence of energy interaction between the states or fragments of trajectory of states (dark blue areas) in Fig. 2, states get "frozen". In regard to the system of atmospheric pollution, this means that there is no dissipation in the atmosphere.

Analysis of distribution of energy of interaction between the states of atmospheric pollution on the test interval (Fig. 3), indicates the promptness of detection of recurrent states and the states of the absence of energy interaction. It is clear that the use in the method (4) of smaller length of trajectory fragments (4 counts) will make it possible to make temporary localization of distributions of energy interaction of states more effectively. At the same time, after the interval of the absence of energy interaction of polluted atmosphere, there is an area of significant energy interaction of states. That is why in order to prevent the occurrence of hazardous conditions, it is necessary to reduce hazardous discharges into the atmosphere at the moment that is previous to the interval of the absence of energy interaction.

The results shown in Fig. 4 indicate that for the proposed method, the length of the fragment of the trajectory of states has little effect on the quality of recognition of areas with a low of energy interaction, if the fragment length does not exceed 10 counts. However, there is a better quality of detection of the areas with a high level of energy interaction for a shorter delay.

Thus, the obtained results generally show the effectiveness of the proposed correlation method for operative detection of recurrent states in complex dynamic systems. In case the proposed method is implemented, the threshold and the method for determining the norm that are traditionally required in the RP methods, are not necessary.

The limitations of the work include the need to conduct more detailed studies of the effectiveness of the proposed method on the example of various actual systems of the technical and natural sphere. This is explained by the fact that the results of the experimental test of the efficiency of the proposed method in the research are checked on a particular example of specific atmospheric contaminants, which are not common. That is why the results of the test of the method on the example of other systems and conditions can be consid-ered as a possible direction for the subsequent development of research.

7. Conclusions

1. The correlation method for operative detection of recurrent states in complex dynamic systems at irregular measurements of vector of states was developed. The concepts of correlation were generalized for the case of vectors of states of the trajectory of dynamics of complex systems and assessment of the correlation of the vectors for a fixed length fragment moving along the trajectory and the interpretation of its magnitude as corresponding degree of energy interaction of the vectors of states for subsequent detection of recurrence of their states. The method is based on the improvement of the phase space of vectors of states by introducing the operation of the scalar product of the vectors. This made it possible to link the level of recurrence of vectors of states in this space to the magnitude of correlation and interpret the magnitude of correlation as a degree of energy interaction of vectors of states. The assessment of correlation and, respectively, the degree of energy interaction between vectors of states of the trajectory of complex dynamic systems is performed only based on the measurements of the vector of states available by the current moment. In this case, it is not required to determine the threshold and the norm traditionally used in the methods of recurrent plots. In addition, the display authenticity requires neither agreement of the norm with the threshold, nor adaptation of the threshold to measurements of states.

2. The performance of the proposed correlation method for operative detection of recurrent states in complex dynamic systems was tested on a particular example of the experimental measurements of the dynamics of the vector of states of exceeding the MAC by pollutants of urban atmosphere. Gas pollutants in the form of formaldehyde, ammonia and carbon monoxide were considered as components of the vector of states. The obtained results generally prove the effectiveness of the proposed method for operative detection of recurrent states in complex dynamic systems. It was experimentally established that the proposed method at irregular measurements of the vector of the states of exceeding the MAC of atmospheric pollution ensures the authentic detection of recurrent states. At the same time, assessment of correlation functions should be made on the territory fragment moving along the trajectory of the vector of states, the length of which should not exceed 10 counts.

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