

*The object of this study is the process of liquid combustion in the pool, and the subject of the study is the characteristics of the random process that describes the heat flow by radiation. Such, in particular, are the law of distribution, mathematical expectation, and correlation function. An experimental study of the combustion of used motor oil in a pool measuring  $9.5 \times 8.7 \text{ m}^2$  was carried out. The mathematical expectation and variance of the cross-sectional area of the flame were determined by video recording followed by the analysis of individual frames. Testing of the hypothesis about the normal law of distribution of the cross-sectional area showed that with a confidence probability of 0.95, the proposed hypothesis does not contradict the experimental data. A selective correlation function and its approximation in the form of  $\sigma^2 \exp(-a|\tau|)$  were constructed. Due to the linear relationship between the cross-sectional area and the heat flux by radiation from the fire, the latter will also have a normal distribution law. At the same time, the magnitude of pulsations (the ratio of the rms deviation to the mathematical expectation) for these random processes will be the same. The value of the parameter  $a$  of the correlation function will also be the same.*

*Taking into account the inertial properties of the device for measuring the heat flux density, a comparison of the experimentally measured values of the heat flux density with the calculated ones was carried out. The measurement results fall into the intervals corresponding to the confidence probability of 0.95. At the same time, the maximum deviation between calculated and experimental data is 14%. From a practical point of view, the built stochastic model opens up possibilities for taking into account random flame pulsations when determining safe zones for the location of personnel and equipment. The model can be used to specify the thermal effect of fire on steel and concrete structures*

**Keywords:** flammable liquid spill, spill fire, stochastic model, heat flow

# CONSTRUCTION OF THE STOCHASTIC MODEL OF THERMAL RADIATION FROM A FLAMMABLE LIQUID SPILL FIRE

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## 1. Introduction

A significant number of emergency situations arising during the transportation, processing, and storage of oil and oil products begin with an emergency liquid spill [1]. The greatest danger is its ignition. This creates a threat of fire spreading to neighboring natural landscapes [2] and technological objects. Heating the steel walls of tanks with petroleum products to the self-ignition temperature of liquid vapors can lead to an explosion of the vapor-air mixture or to the combustion of vapors at the outlet of breathing devices. The situation of the first type occurs if the concentration of vapors in the gas space of the tank is between the lower and upper concentration limits of flame propagation. The situation of the second type occurs if the concentration of vapors in the gas space of the tank exceeds the upper concentration limit of flame propagation. Also, the thermal effect of fire leads to the loss of strength of steel structures [3] and depressurization of flange connections. Another consequence of such accidents is the pollution of underground and river waters due to seepage of liquid deep into the soil [4] and the release of hazardous substances into the atmosphere [5]. Spreading over considerable distances, they significantly affect the state of the air and create risks for the population [6].

Despite safety measures, accidents involving the spilling and burning of flammable liquids still occur. This is con-

firmed by emergency situations related to the spillage or burning of flammable liquids that have occurred recently:

- 2023 (USA, Connecticut). As a result of the accident of the gasoline truck, about  $8.3 \text{ m}^3$  of fuel spilled and caught fire;
- 2021 (USA, Texas). A train with petroleum products derailed and collided with a truck. 3 cisterns caught fire, the height of the flames from the fire was several tens of meters. Residents of nearby houses were evacuated;
- 2020 (Great Britain, Llangennech). A freight train carrying diesel fuel derailed and caught fire, spilling the fuel into a nearby river. About 300 residents of nearby houses were evacuated;
- 2020 (Kazakhstan, Zhambyl region). A gasoline tank derailed, resulting in a spill and fire. The area of the fire was about  $600 \text{ m}^2$ ;
- 2016 (China, Jingjiang). The fire in the pipelines caused a large amount of oil to leak and catch fire. The area of the fire was about  $2000 \text{ m}^2$ .

In this regard, there is a need to conduct research aimed at determining the heat flow from fires and spills of flammable liquids.

## 2. Literature review and problem statement

Paper [7] provides an overview of large-scale accidents associated with the spillage of flammable liquids. Empirical

and semi-empirical models of liquid spreading on water and on a solid surface are considered. But the models of liquid combustion and the thermal effect of fire on neighboring objects are left out of consideration.

In [8], the practical significance of studies of liquid combustion in pools is indicated – situations where the spread of flammable liquid is limited by obstacles. Examples of pool fires are the burning of petroleum products in tanks or spills. The flame structure was analyzed, and several characteristic zones were identified. Above the surface of the liquid is a zone saturated with flammable vapors. It is a constant visible flame of a conical shape. Above is a zone of bright intermittent flame, in which air enters the combustion zone radially. Above the flame is a non-reactive zone of smoke, which is usually completely turbulent in nature. But flame pulsations caused by the intermittent flame zone are not investigated in the work.

In [9] there is an overview of studies on the combustion of petroleum products in basins. Flame pulsations, specific mass burning rate, flame height, heat transfer by radiation are considered. But a significant limitation of these studies is the condition of the absence of wind.

In [10], the dependence of pulsations on the dimensions of the combustion cell is studied. It is shown that it is possible to distinguish global pulsations, the frequency of which is a function of the diameter of the combustion cell. But at the same time, the random component of pulsations remains neglected. In [11], the frequency of flame pulsations during combustion near the wall and in the corner was additionally investigated. But even here, the random component of pulsations is not investigated.

In [12], time series are used to build a stochastic model of thermal radiation, and their parameters are determined on the basis of an experiment with liquid combustion in a container with a diameter of 7.1 cm. But the combustion regime of liquids in small-diameter containers is significantly different from the combustion regime in tanks with a diameter of more than 1 m. In [13], such characteristics of flame pulsations during burning in the openings of ventilated facades as the average value of pulsations (the ratio of the maximum flame length to the average length) and the frequency of pulsations are studied. But the parameters that describe the random component of pulsations are not considered.

In [14], the thermal effect of fire on steel structures was considered, but the parameters of the combustion center and the dynamics of heat flow changes were neglected. In [15], a model of the thermal effect of a fire spilling a flammable liquid on a nearby oil product tank was built. The model takes into account heat transfer by radiation and convection. But the shape of the torch is assumed to be constant. In [16], water jet cooling of a tank heated under the thermal influence of a fire is considered, but here, too, a constant heat flow from the combustion center is considered. In [17], the convection component of the heat flow from a spill fire of arbitrary shape was considered in detail, the influence of flame pulsations on the heat flow was neglected. In [18], it was noted that the traditional idea of a flame in the form of a cylinder or a cone can lead to an error in the calculation of the heat flow, and a refinement was proposed by adjusting the diameter and height of the flame. But the shape of the flame is considered constant, random pulsations are not taken into account. In [19], an experimental study of the rate of mass loss, flame height, degree of its blackness, and thermal radiation density was carried out for combustion

cells (0.1÷0.4) m. However, extrapolation to combustion cells with a larger diameter is complicated due to the increased influence of turbulence on combustion process. In [20], the deformation of the tank walls under the thermal influence of a fire in the tank is considered. It is noted that the pulsations of the torch have a significant effect on the heat flow. But the parameters of the random component of pulsations have not been investigated. In [21], a model of tank wall heating under the thermal influence of a fire in a nearby tank was built, in which a normal distribution of the mutual radiation coefficient is assumed, but the parameters of the distribution are assumed to be given a priori.

Therefore, the calculation of the heat flow from a combustible liquid spill fire requires the determination of the geometric shape of the flame [15]. But due to the turbulent nature of liquid combustion, the shape of the flame above the spill is not constant [18]. Random pulsations of the flame lead to random changes in the mutual radiation coefficient [21]. This, in turn, leads to the random nature of the density of the heat flow falling on neighboring objects.

Thus, the random component of the heat flow from the fire due to the random pulsations of the flame, which is caused by the turbulent mode of combustion, has not been considered in the available studies. This gives reason to assert that it is expedient to conduct research aimed at building a stochastic model of heat radiation from a fire, which will take into account the random nature of flame pulsations.

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### 3. The aim and objectives of the study

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The purpose of this work is to build a stochastic model of thermal radiation from a combustible liquid fire. In practice, this opens up opportunities for calculating the thermal impact on nearby technological objects, as well as determining safe zones for the location of personnel and equipment involved in fire suppression.

To achieve this goal, the following tasks must be solved:

- to determine the characteristics of the flame, which describe its random pulsations;
- to determine the characteristics of a random process describing the heat flow by radiation from a fire;
- to conduct an experimental verification of the thermal radiation model from a combustible liquid fire.

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### 4. The study materials and methods

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The object of this study is the process of liquid combustion in the pool, and the subject of the study is the characteristics of the random process that describes the heat flow by radiation. Such, in particular, are the law of distribution, mathematical expectation, and correlation function. The main hypothesis of the research assumes that heat flow by radiation can be represented as a random process with a normal distribution law and a certain correlation function. The main assumption is ergodicity and stationarity of the random process.

To determine the characteristics of random flame pulsations over a liquid spill, an experimental study of the combustion of used engine oil in a rectangular basin measuring 9.5×8.7 m<sup>2</sup> was conducted. The burning process was filmed using a Canon (Japan) PowerShot A710 IS camera. The video recording was divided into separate frames (25 frames

per second recording), and the number of pixels belonging to the flame was determined in each frame. The number of pixels was converted into a cross-sectional area based on the number of pixels corresponding to the length of the pool. A BP-2 bolometer (Ukraine) was used to measure the heat flux density. At the same time, the bolometer was directed to the visual middle of the flame. Measurements were consistently carried out at pre-selected points that were marked on the terrain. Mathematical statistics methods were used to estimate distribution parameters and test the hypothesis about the distribution law. The fastest descent method was used to estimate the parameters of the correlation function. Control theory methods were used to take into account the influence of random pulsations when measuring the heat flux density.

**5. Results of building a stochastic model of thermal radiation from a combustible liquid fire**

**5.1. Determination of flame characteristics describing its random pulsations**

After the ignition and setting of the liquid combustion process to the established mode, the video recording of the process was carried out (Fig. 1), which ended with the start of extinguishing. The duration of the video recording was about 3 minutes.



Fig. 1. Combustion of used engine oil in the pool

The frequency of global (non-random) flame pulsations can be estimated using the following formula [10]:

$$f = 0.53 \sqrt{\frac{g}{D_{eff}}}, \tag{1}$$

where  $g$  is the acceleration of gravity;  $D_{eff}$  is the effective diameter of the combustion chamber. In particular, for a rectangle, the effective diameter is determined by the formula:

$$D_{eff} = \frac{2a + 2b}{\pi},$$

where  $a, b$  are the sides of the rectangle.

It follows from (1) that the frequency of global flame pulsations for a pool of  $9.5 \times 8.7 \text{ m}^2$  is approx-

imately equal to 0.48 Hz. This corresponds to a pulsation period of 2.1 s. Therefore, the study interval contained about 90 complete periods. The cross-sectional area was determined for each video recording frame. For this, the procedure given in [22] was used. Its essence consists in determining the number of points belonging to the flame, and their subsequent conversion into the area of the flame.

Fig. 2 shows a fragment of observations on the cross-sectional area. The average value, the root mean square deviation, and the magnitude of pulsations of the cross-sectional area were, respectively:

$$\bar{S} = 22.93 \text{ m}^2; \sigma_s = 4.62 \text{ m}^2;$$

$$\delta = \sigma_s / \bar{S} \approx 0.2.$$

The correlation function  $K_\xi(\tau)$  of the stationary random process  $\xi(t)$  takes the form

$$K_\xi(\tau) = M[\xi(t + \tau) - \bar{\xi}][\xi(t) - \bar{\xi}].$$

The estimate  $\hat{K}_\xi$  of the correlation function  $K_\xi$  of the random process  $\xi(t)$  based on its observations  $x(t)$  at discrete moments of time  $0, \Delta t, 2\Delta t, \dots, n\Delta t$  takes the form

$$\hat{K}(m\Delta t) = \frac{1}{n-m} \sum_{i=0}^{n-m} [x(i\Delta t) - \bar{x}][x((i+m)\Delta t) - \bar{x}], \tag{2}$$

where  $\bar{x}$  is the sample mean.

Fig. 3 shows the estimate of the correlation function (2) for the cross-sectional area of the flame, as well as its approximation in the form

$$\tilde{K}(\tau) = \sigma^2 \exp(-\alpha|\tau|), \tag{3}$$

where the values of parameters  $\sigma$  and  $\alpha$  were determined by the method of least squares:

$$\sum_m [\hat{K}(m\Delta t) - \sigma^2 \exp(-\alpha|m\Delta t|)]^2 \rightarrow \min. \tag{4}$$

The application of the method of fastest descent to the minimization problem (4) gave  $\sigma^2 = 20.50 \text{ m}^4; \alpha = 1.21 \text{ s}^{-1}$ :

$$\tilde{K}(\tau) = 20.50 \exp(-1.21|\tau|). \tag{5}$$

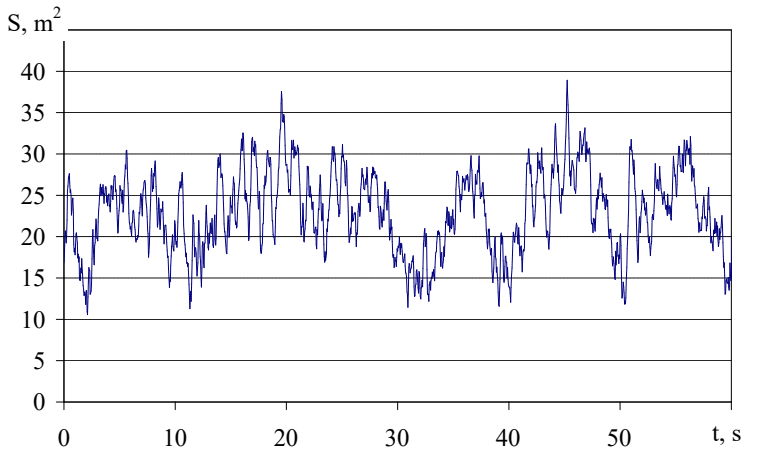


Fig. 2. Change in the cross-sectional area of the flame over time during the combustion of used engine oil in a rectangular pool

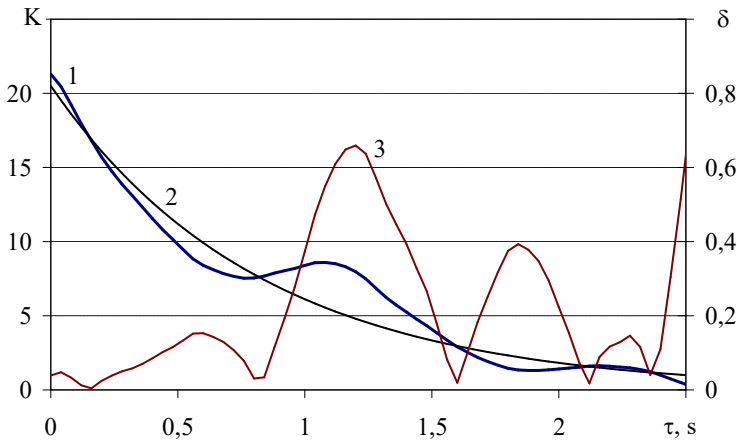


Fig. 3. Correlation function of the cross-sectional area of the flame during combustion of used engine oil in a rectangular basin:  
 1 – estimate  $\hat{K}$  obtained from (2); 2 – approximation  $\tilde{K}$ ;  
 3 – relative approximation error (on the right axis)

Testing the hypothesis about the distribution law of a random variable requires a sample of independent variables. At the same time, the results of observations of the cross-sectional area at successive moments of time (interval between observations  $\Delta t=0.04$  s) correlate well (Fig. 2). Therefore, observations at points  $t=0, 2, 4, \dots$  s were chosen to test the hypothesis. A step of 2 s corresponds to a correlation coefficient of 0.07 between successive values of a random variable. Considering this correlation to be insignificant, the hypothesis about the distribution law of the random variable  $\xi$  was tested. The numerical axis was divided into seven intervals and distribution histograms were constructed (Fig. 4).

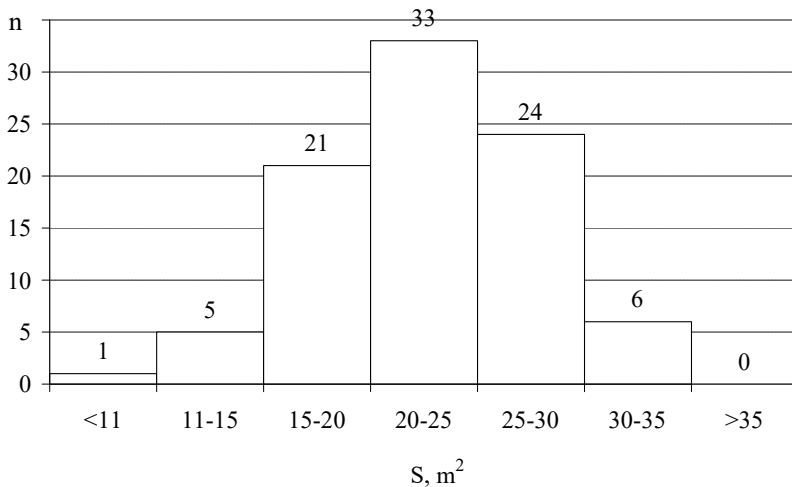


Fig. 4. Histogram of the cross-sectional area distribution during the combustion of used engine oil in a rectangular pool

Testing the hypothesis about the normal distribution law using the  $\chi^2$  criterion showed that the calculated value of the  $\chi^2$  value is:

$$\chi^2_{pool} = \sum_{i=1}^7 \frac{(n_i - Np_i)^2}{Np_i} \approx 1.32,$$

where  $N$  is the sample volume;  $n_i$  is the experimental frequency of value falling into the  $i$ -th interval;  $p_i$  is the theoretical

probability of falling into the  $i$ -th interval. At the same time, the tabular value of  $\chi^2$  with four degrees of freedom and a confidence probability of 0.95 is:

$$\chi^2_4(0.95) = 9.49.$$

Since the calculated value turned out to be less than the tabular value, with a confidence probability of 0.95, the hypothesis about the normal distribution of the cross-sectional area of the torch does not contradict the experimental data.

Given that the cross-sectional area corresponds to the solid angle at which the flame is visible from a given point, the ratio of the root-mean-square deviation to the mean value is the same for the cross-sectional area and the mutual radiation coefficient:

$$\delta = \frac{\sigma_\psi}{\bar{\psi}} = \frac{\sigma_S}{\bar{S}} \approx 0.2. \tag{6}$$

At the same time, the coefficient of mutual irradiation will also have a normal law of distribution and a correlation function of the form:

$$K_\psi(\tau) = \sigma_\psi^2 \exp(-\alpha|\tau|),$$

in which the value of the parameter  $\alpha$  coincides with the corresponding value of the correlation function of the cross-sectional area.

### 5. 2. Characterization of a stochastic process describing the heat flux by radiation from a fire

The representation of the heat flow from a fire as a random process requires the determination of the distribution law, distribution parameters (mathematical expectation and variance), as well as the correlation function.

The heat flux density from a fire is determined by the Stefan-Boltzmann law:

$$q = c_0 \varepsilon_f \left[ \left( \frac{T_f}{100} \right)^4 - \left( \frac{T}{100} \right)^4 \right] \psi,$$

where  $c_0=5.67$  W/(m<sup>2</sup>·K<sup>4</sup>) is constant;  $\varepsilon_f$  is the degree of blackness of the emitting surface of the torch;  $T_f$  is the temperature of the torch surface;  $T$  is the temperature of the elementary site on which thermal radiation falls;  $\psi$  is the mutual radiation coefficient between the torch and this elementary platform.

Given the random nature of the mutual radiation coefficient caused by flame pulsations, the heat flux density  $q(t)$  is also a random process. Due to the linear relationship between the heat flux density  $q(t)$  and the mutual radiation coefficient  $\psi(t)$ , their mathematical expectations and root mean square deviations are related by the following relations:

$$\bar{q} = c_0 \varepsilon_f \left[ \left( \frac{T_f}{100} \right)^4 - \left( \frac{T}{100} \right)^4 \right] \bar{\psi}; \tag{7}$$

$$\sigma_q = c_0 \varepsilon_f \left[ \left( \frac{T_f}{100} \right)^4 - \left( \frac{T}{100} \right)^4 \right] \sigma_\psi. \quad (8)$$

Then the values of pulsations of the cross-sectional area of the flame and the density of the heat flux coincide:

$$\frac{\sigma_q}{\bar{q}} = \frac{\sigma_\psi}{\bar{\psi}} = \frac{\sigma_S}{\bar{S}} = \delta. \quad (9)$$

To calculate the mathematical expectation of the heat flux density, it was assumed that the average flame length is described by the following expression [23]:

$$L = bD \left( \frac{\eta}{\rho_a \sqrt{gD}} \right)^n (\omega^*)^{-0.21}; \quad (10)$$

$$\cos \alpha = (\omega^*)^{-0.5},$$

where  $b=55$ ;  $n=0.67$ ;  $\eta$  – specific mass burning rate;  $\rho_a$  – air density;  $g$  – acceleration of gravity;  $D$  – diameter of the filling;  $\alpha$  – angle of inclination of the flame relative to the vertical axis under the influence of the wind;  $\omega^*$  – dimensionless wind speed:

$$\omega^* = \max \left\{ 1, \omega \left( \frac{g\eta D}{\rho_a} \right)^{-1/3} \right\};$$

$\omega$  is wind speed.

To determine the average flame length at an arbitrary point  $(x, y)$  on the liquid surface, a modification of formula (10) was used in the form:

$$z(x, y) = 2bR \left( \frac{\eta}{\rho_a \sqrt{2gR}} \right)^n (\omega^*)^{-0.21}, \quad (11)$$

where  $R=R(x, y)$  is the distance from the point  $(x, y)$  to the edge of the pool. This has made it possible to determine the average value of the mutual radiation coefficient:

$$\bar{\psi} = \frac{1}{\pi} \iint_S \frac{\cos \phi_1 \cos \phi_2}{r^2} dS, \quad (12)$$

where  $r$  is the radius vector connecting the point on the flame surface and the elemental platform on which the heat flow falls;  $\phi_1$  is the angle between the normal vector to the elementary site and the radius vector  $r$ ;  $\phi_2$  is the angle between the normal vector to the flame surface and the radius vector  $r$ . At the same time, the integration in (12) is performed only over that part of the flame surface where  $\cos \phi_1 > 0$ ,  $\cos \phi_2 > 0$ .

Substitution (12) in (7) makes it possible to determine the mathematical expectation of the heat flux density. Then its root mean square deviation can be found from (9):

$$\sigma_q = \delta \bar{q}. \quad (13)$$

The stationarity of the random process  $\psi(t)$  also implies the stationarity of the random process  $q(t)$ . Then the correlation function of the heat flux density differs from the correlation function of the mutual radiation coefficient only by a factor:

$$K_q(t) = \sigma_q^2 e^{-\alpha|t|}. \quad (14)$$

Stationary random processes are characterized by the spectral density  $S(\omega)$ , which is related to the correlation function by the Fourier transform:

$$S(\omega) = \int_{-\infty}^{\infty} K(\tau) e^{-j\omega\tau} d\tau; \quad (15)$$

$$K(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega\tau} d\omega,$$

where  $j = \sqrt{-1}$  – imaginary unit. For the correlation function of form (14), the spectral density is:

$$S_q(\omega) = \frac{2\sigma_q^2 \alpha}{\omega^2 + \alpha^2}. \quad (16)$$

As a result of the linear relationship between the random processes  $q(t)$  and  $\psi(t)$ , the normal distribution law of the process  $\psi(t)$  follows from the normal distribution law for the random process  $q(t)$ .

### 5. 3. Experimental verification of the model of thermal radiation from a flammable liquid fire

Since the heat flow is a random function of time, and the device for its measurement has inherent inertia, the value obtained as a result of measurements is the result of averaging over some time interval. This leads to the fact that the variance of the measurement result will differ from the variance of the input heat flow. From the point of view of the theory of automatic control, the measuring device can be considered as an aperiodic link of the first order. The transfer function of such a link takes the form:

$$W(p) = \frac{K}{\tau p + 1}, \quad (17)$$

where  $K$  is the transmission coefficient;  $\tau$  is the time constant characterizing the inertial properties of the link. The work of the link can be represented by the diagram shown in Fig. 5.

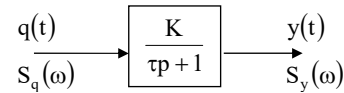


Fig. 5. Representation of the bolometer as a first-order aperiodic link

A heat flow is received at the bolometer input, which is described by a random stationary process  $q(t)$  with a spectral density  $S_q(\omega)$ . At the output, we get the measurement result as a stationary random process with spectral density  $S_y(\omega)$ .

The spectral densities of the initial and resulting random processes are related by the ratio:

$$S_y(\omega) = |W(j\omega)|^2 S_q(\omega). \quad (18)$$

Calculating the square of the frequency response modulus, we get:

$$\begin{aligned} |W(j\omega)|^2 &= \left| \frac{K}{1 + j\omega\tau} \right|^2 = \left| \frac{K(1 - j\omega\tau)}{1 + \omega^2\tau^2} \right|^2 = \\ &= K^2 \frac{1 + \omega^2\tau^2}{(1 + \omega^2\tau^2)^2} = \frac{K^2}{1 + \omega^2\tau^2}. \end{aligned} \quad (19)$$

After substituting expressions (16) and (19) into (18), we get:

$$S_y(\omega) = \frac{K^2}{1 + \omega^2\tau^2} \frac{2\sigma_q^2\alpha}{\omega^2 + \alpha^2} \quad (20)$$

Given that the variance of a stationary random process is related to the spectral density by the expression:

$$\sigma_y^2 = \frac{1}{\pi} \int_0^\infty S_y(\omega) d\omega,$$

we have:

$$\begin{aligned} \sigma_y^2 &= \frac{2\sigma_q^2\alpha K^2}{\pi} \int_0^\infty \frac{d\omega}{(1 + \omega^2\tau^2)(\omega^2 + \alpha^2)} = \\ &= \frac{2\sigma_q^2\alpha K^2}{\pi} \frac{\pi}{2\alpha(\tau\alpha + 1)} = \sigma_q^2 \frac{K^2}{\tau\alpha + 1}. \end{aligned}$$

Taking the transfer coefficient  $K=1$ , the following is finally obtained:

$$\sigma_y^2 = \sigma_q^2 \frac{1}{\tau\alpha + 1} \quad (21)$$

From the analysis of dependence (21), it follows that the measurement result using a bolometer has a smaller variance than the heat flux density by radiation, which falls on the sensitive element of the bolometer. This reduction will be more significant the greater the inertia of the measuring device and the less correlated the input signal.

For example, for the BP-2 bolometer, the value of the parameter  $\tau=4$  s. Then the coefficient of reduction of the variance of the signal  $y(t)$  takes the form:

$$k_y = \frac{\sigma_y^2}{\sigma_q^2} = \frac{1}{4\alpha + 1} \quad (22)$$

In particular, for the value of the parameter  $\alpha$  of the correlation function obtained for the heat flux density from the combustion center in the pool, the variance reduction factor is 0.17.

To check the adequacy of the model of thermal radiation from a combustible liquid fire, a comparison of calculated values with experimental ones was carried out.

The heat flux density was measured using a bolometer. The points at which the measurements were made were selected in advance based on the calculations of the heat flux density (Fig. 6). They were marked with flags on the ground. Points are bypassed in sequence, and in each of them the bolometer was directed to the middle of the flame. The wind was directed as shown in Fig. 5 and had an average speed of 2 m/s, ambient air temperature –  $T_0=19$  °C. The degree of blackness of the flame was taken as  $\epsilon_f=0.85$ . The root mean square deviation of the heat flux density at point  $i$  was assumed equal to:

$$\sigma_i = \delta\sqrt{k_y}q_i, \quad (23)$$

where  $\delta=0.2$  (according to (6));  $k_y=0.17$  (according to (22));  $q_i$  is the calculated value of the heat flux density. Then:

$$\sigma_i = 0.082q_i.$$

Taking into account the normal distribution law for the heat flux density, with a confidence probability of 0.95 the value  $\hat{q}_i$  measured by the bolometer should be within the range of  $\pm 2.5\sigma$ :

$$q_i - 2.5\sigma_i < \hat{q}_i < q_i + 2.5\sigma_i. \quad (24)$$

A comparison of the calculated values of the heat flux density and the result of the experiment is shown in Fig. 7.

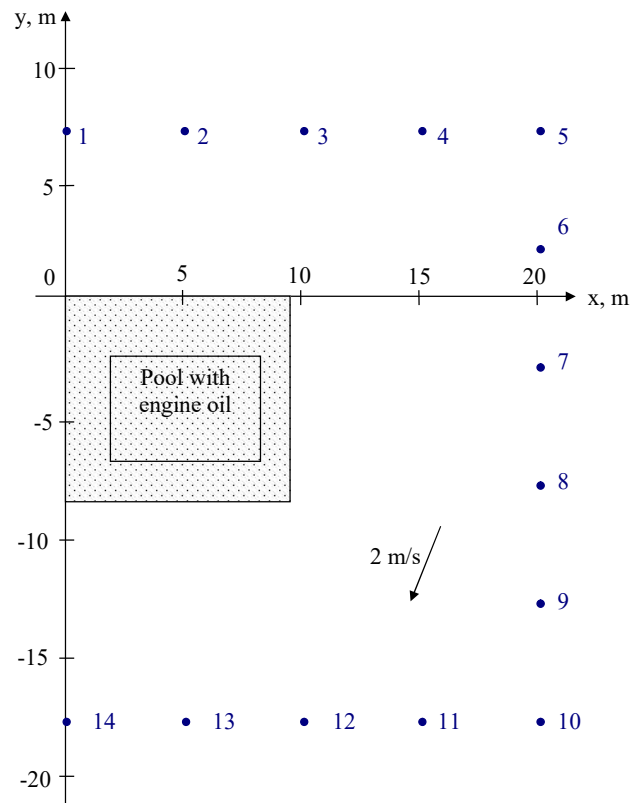


Fig. 6. Scheme of the experiment with the combustion of used motor oil in the basin: 1–14 – points where the heat flux density was measured

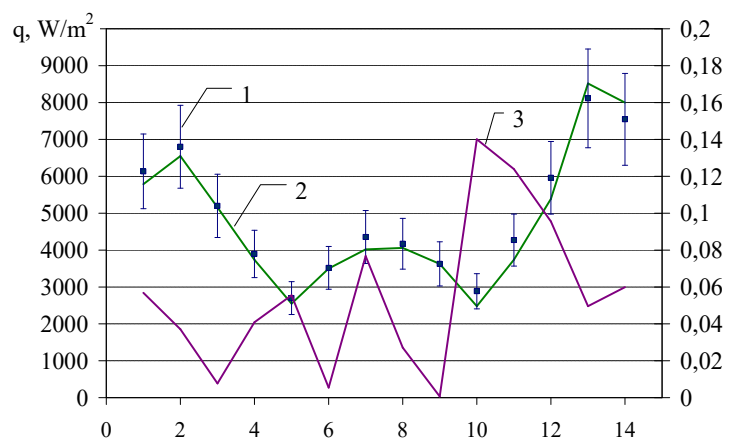


Fig. 7. Comparison of calculated values of heat flux density with experimental data: 1 – confidence intervals for the constructed model of heat radiation from a rectangular spill; 2 – experimental values; 3 – relative deviation between calculated data and experimental results (on the right axis)

Analysis of the results shown in Fig. 7 reveals that the experimental data fall into confidence intervals of  $\pm 2.5\sigma$ , while the maximum deviation between the calculated and experimental data is 14 %.

## 6. Discussion of results of building a stochastic model of thermal radiation from a flammable liquid fire

Video recording of the combustion process of used engine oil in the pool with further processing of the frames allowed us to obtain the realization of a random process corresponding to the cross-sectional area of the flame (Fig. 2). Visual observation of the flame shows that its shape is not constant, the flame is characterized by random pulsations (Fig. 1). The burning of engine oil is accompanied by thick black smoke, which hides part of the flame. But smoke particles are not an obstacle for thermal radiation, which lies in the infrared part of the spectrum. This means that although the given approach has an error in estimating the mathematical expectation and the root mean square deviation of the cross-sectional area, it can be used to estimate their ratio  $\delta$ , which characterizes the magnitude of pulsations. The resulting realization of the random process  $\xi$ , which describes the cross-sectional area of the flame, allows us to get an estimate of its correlation function according to formula (2). Analysis of graphical dependences in Fig. 2 shows that the selective correlation function is satisfactorily approximated by a function of the form  $\sigma^2 \exp(-\alpha|\tau|)$ . Estimates of parameters  $\sigma^2$  and  $\alpha$  were obtained as a solution to the minimization problem (4). We used the fastest descent method to solve it. The error plot (Fig. 3) has “bumps” with a step of about 0.55 s, which corresponds to the periodic component in the flame pulsations with a frequency of about 0.9 Hz. This value coincides in magnitude with estimate (1) [10].

The resulting implementation of the random process (Fig. 2) also makes it possible to determine its distribution law. Shown in Fig. 4, the histogram of the distribution of the cross-sectional area of the flame gives reason to put forward the hypothesis of a normal distribution law. Hypothesis testing using the  $\chi^2$  test shows that with a confidence probability of 0.95, the hypothesis of a normal distribution does not contradict the experimental data. The cross-sectional area corresponds to the solid angle at which the flame is visible from a given point. Then the conclusions regarding the distribution law, the magnitude of pulsations  $\delta$  and the type of correlation function made for the cross-sectional area will also be true for the mutual radiation coefficient.

Modeling the heat flow from a fire by a random process requires the determination of the distribution law, distribution parameters, as well as the correlation function. As a result of the linear relationship between the coefficient of mutual irradiation and the radiation heat flux density, the distribution law for the heat flux density will be normal. The mathematical expectation and root mean square deviation are given by formulas (7) and (8), respectively. To calculate the mathematical expectation of the heat flow density, model (10) [23] was chosen, which describes the length of the flame above the spill in the form of a circle in the presence of wind and is characteristic of a wide class of combustion sites. In order to construct the radiating surface of the flame above the rectangular basin, an assumption was made about the length of the flame (11) at a given point. The correlation function of the heat flux density has a form that is similar

to the correlation function of the cross-sectional area of the flame. At the same time, the values of the  $\alpha$  parameter of these functions coincide. Applying the Fourier transform to the correlation function (14) has made it possible to obtain the spectral density (15) of the random process.

Since the heat flux density is a function of time, and the device for its measurement has a certain inertia, the result of the measurement will not be the instantaneous value of the quantity but averaged over some interval. From the point of view of the theory of automatic control, any device in the first approximation can be represented as an aperiodic link of the first order (Fig. 5), the transfer function of which takes the form (17). This makes it possible to obtain the spectral density of the signal (20) at the output of the bolometer and calculate its variance (21). From the analysis of expression (21), it follows that the variance of the signal at the output of the bolometer will always be smaller than the variance of the heat flux density at the input. Moreover, the attenuation coefficient (22) will be all the more significant, the greater the inertia of the device and the lower the correlation of the heat flow.

During the experiment, the heat flux density was measured at preselected points (Fig. 6). A comparison of the calculated data with the experiment (Fig. 7) shows that the measurement results at all points fall within the confidence intervals of  $\pm 2.5\sigma$ , which corresponds to a confidence probability of 0.95. At the same time, the maximum deviation between calculations and experimental data is 14 %.

The advantage of the model is that it makes it possible to take into account random pulsations of the heat flow by radiation from a liquid spill fire, which are caused by the turbulent mode of the combustion process.

Limitations of the model include the impossibility of its application in the case of non-stationary heat flow. For a stationary heat flow, the application of the model is limited to the range of linear dimensions of the combustion cell from several meters to several tens of meters.

The disadvantage of the built model is that its parameters are determined on the basis of an experiment with the combustion of a certain liquid in a basin of certain dimensions. This complicates its use for other types of combustible liquids and dimensions of the combustion chamber. Thus, the prospects for further research are related to the generalization of the obtained estimates of the magnitude of pulsations and the parameters of the correlation function.

The proposed stochastic model of thermal radiation from a fire can be used to predict the impact on steel and concrete structures [24]. The results could also be used in the design of fire detectors [25].

## 7. Conclusions

1. By conducting an experiment with the combustion of used engine oil in a rectangular basin with dimensions of  $9.5 \times 8.7$  m<sup>2</sup>, a random process describing the cross-sectional area of the torch was investigated. To do this, a video recording of the burning process was carried out, the number of pixels corresponding to the flame was determined, and their conversion into an area was carried out. The mathematical expectation and variance were estimated based on the obtained realization of the random process. A sample correlation function was constructed, and its approximation was carried out by a function of the form  $\sigma^2 \exp(-\alpha|\tau|)$ , where

$\alpha=1.21$ , and the magnitude of pulsations (the ratio of the root mean square deviation to the mathematical expectation) is 0.2. The hypothesis about the normal distribution law of the cross-sectional area was put forward and it is shown that with a confidence probability of 0.95 the hypothesis does not contradict the experimental data.

2. Due to the fact that the cross-sectional area of the torch is proportional to the mutual irradiation coefficient, the normal distribution law for the mutual irradiation coefficient follows from the normal distribution law of the cross-sectional area. This, in turn, leads to a normal distribution law for the heat flux density from the fire. The assumption of the “average” shape of the torch made it possible to find a mathematical expectation of the heat flux density. It is shown that the magnitude of the pulsations of the heat flux density coincides with the magnitude of the pulsations of the cross-sectional area. This has made it possible to determine the variance of the heat flux density. It is shown that the correlation function of the heat flux density has the same value of the parameter  $\alpha=1.21$  as the correlation function of the cross-sectional area.

3. The variance of the signal at the output of the device for measuring the heat flux density was determined. It is shown that the variance of the signal at the output of the device would always be smaller than the variance of the heat flux density at its input. This decrease will be all the more

significant, the larger the value of the parameter  $\alpha$  of the correlation function of the input signal. In particular, under the conditions of the experiment, the attenuation coefficient was 0.17. Comparison of the calculated data with the experiment shows that the measurement results fall into the intervals corresponding to the confidence probability of 0.95. At the same time, the maximum deviation between calculated and experimental data is 14 %.

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#### Conflicts of interest

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The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study and the results reported in this paper.

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#### Data availability

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The data will be provided upon reasonable request.

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