- APPLIED MECHANICS -

DEVISING AN ENGINEERING PROCEDURE FOR CALCULATING THE DUCTILITY OF A ROLLER BEARING UNDER A NO-CENTRAL RADIAL LOAD

Відомі теоретичні підходи до розрахунку податливості підшипників кочення містять досить складні аналітичні залежності і вимагають громіздких обчислень. Тому дослідження, яке спрямоване на розробку інженерного підходу до розрахунку радіальної податливості підшипників, є актуальним.

У цьому дослідженні запропонована інженерна методика визначення радіальної податливості на прикладі циліндричних роликових підшипників. Враховано радіальний зазор, контактні деформації деталей, деформації вигину і перекіс кілець для випадків дії на підшипник центрального радіального навантаження і радіального навантаження з ексцентриситетом. Прийнята спрощена лінійна розрахункова модель визначення кута перекосу кілець справедлива для малих кутів, коли зберігається контакт по всій довжині ролика. Розрахунок радіальної податливості роликових підшипників при не центральному радіальному навантаженні ґрунтується на визначенні суми змінних пружних деформацій в контакті кілець і найбільш навантаженого ролика. Значення пружних деформацій визначаються за відомими формулами рішення контактної задачі теорії пружності з урахуванням розбіжності геометричних центрів зовнішнього і внутрішнього кілець.

Адекватність запропонованої методики підтверджується результатами розрахунків питомої податливості циліндричного роликового підшипника 2211 з центральним радіальним навантаженням. За запропонованою методикою отримані значення питомої податливості на 3...4 % менші в порівнянні з аналогічними результатами відомої методики. На прикладі циліндричного роликового підшипника 42726 проведено дослідження конструктивних параметрів з урахуванням не центрального радіального навантаження. Показано зниження податливості підшипника 42726 зі збільшенням кількості роликів і жорсткості зовнішнього кільця, а також зі зменшенням ексцентриситету радіального навантаження.

Податливість підшипників кочення необхідно знати при складанні динамічних моделей деяких машин: шпинделів верстатів, валів-шестерень великогабаритних редукторів, кранових конструкцій. Тому запропонована інженерна методика визначення податливості роликових підшипників при малих кутах перекосів кілець може знайти застосування в практиці проектування машин і механізмів, де важливі пружні характеристики всіх їх елементів

Ключові слова: інженерна методика розрахунку, податливість роликових підшипників, контактні деформації, не центральне радіальне навантаження, перекіс кілець

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1. Introduction

The ductility of a roller bearing is defined by the elastic deformations in contacts between the most loaded rolling body and the rings rolling tracks. The ductility of a bearing depends on its type, the number of rolling elements, the contact angle between rolling elements, a material of the parts, gaps, and preload in bearings, the scheme of application and

magnitudes of loads. Typically, the elastic deformations of bearing parts are small and are ignored in the calculation of machines. However, for cases of constructing the dynamic models of spindles for machine tools, shaft-gears for large-size reducers, as well as crane structures, the ductility of bearings is an important factor. In addition, the deformation of a shaft's bearings, such as a gearbox, under the influence of support reactions that are different in magnitude leads to a relative misalignment of conjugated toothed gears that shortens their service life. Therefore, it is a relevant task for the calculation of modern machines and mechanisms to undertake a research aimed at developing an engineering procedure for determining the radial ductility of bearings with different schemes of their loading.

2. Literature review and problem statement

Calculation methods for radial ductility of rolling bearings differ in approaches and models to representing the contact areas. The ductility of ball bearings, as proposed in [1], should be calculated taking into consideration the assumption on the permanence of contact angle based on the simplified analytical method by using an iterative Newton's algorithm. That has made it possible to perform, without the use of specialized software, the required computation of ball bearings ductility. However, there is no information that would account for the radial gap and the deformation of bearing rings.

Paper [2] suggests calculating the ductility of spherical roller bearings by using known formulae for determining elastic deformation and load on the central rolling body without taking into consideration features of the design. The working formulae to calculate the ductility of bearings are cumbersome and are not convenient to practical calculations.

A study into the influence of dimensions of different types of bearings on their ductility was reported in [3]; the author used known formulae for the contact problem in the theory of elasticity. In this case, he did not take into consideration the influence of bearings gaps, the flexural deformation of rings, and the misalignment of rings.

An analytical method was proposed in [4] to calculate the ductility of roller bearings. However, the suggested method does not take into consideration the impact of a radial gap, the possibility of flexural deformation, and the misalignment between the rings of a bearing.

A study into the influence of a combined (radial and momentum) load on the ductility of cylindrical roller combined bearings was reported in paper [5] based on a model that takes into consideration four degrees of freedom. The combined effect of radial and moment loads predetermined the misalignment of rings and the non-linear character of ductility. However, the authors of [5], despite the complexity of their model, did not consider a no-central application of the outer radial load, which also contributes to forming the misalignment of rings.

A research into the influence of different profiles of rollers, which reduce the rim concentration of contact stresses, on ductility of a bearing was performed in paper [6] by using numerical methods. In this case, no analytical research methods were discussed in that publication.

To carry out an analytical study into the impact of the profile of a cylindrical roller on the radial ductility of a bearing, paper [7] constructed a method for cutting the contact zone by layers. However, the proposed procedure for calculating bearing ductility is cumbersome and rather complicated.

Article [8] suggested an approach to determining the ductility of a bearing based on a finite element method. The authors compared results from their calculations using numerical and analytical methods. The analytical solution accepted is a single contact between a rolling body conjugation and a ring rolling track. However, it is not clear how one could use the proposed procedure to simulate the misalignment of bearing rings.

Paper [9] suggested an energy method for determining the ductility of a bearing. The method is based on the principle of a minimum potential energy from classical mechanics in order to establish the relationship between a load on the bearing and its deformation. Implementation of the method necessitates rather cumbersome calculations, not convenient in engineering practice.

Based on the results from analytical estimation and experimental measurements of the ductility of defective bearings at different magnitudes of a static radial load, as well as the angular positions of a separator, article [10] established a link to the amount of the identified damage. However, it does not give the algorithm for the analytical estimation of bearings ductility, which hampers the practical application of the proposed study.

An analysis of papers [1–10] reveals that known theoretical approaches to calculating the ductility of rolling bearings include rather complicated analytical dependences and require cumbersome computation. At the same time, the authors do not take into consideration the patterns in bearing operation associated with the causes and consequences of forming the misalignment of rings. Remains unaddressed up to now is the simplest case of loading a bearing by a no-centrally applied radial load. Therefore, it is expedient to devise an engineering procedure for calculating the radial ductility of roller bearings in order to study a no-centrally applied radial load, which causes the misalignment of rings.

3. The aim and objectives of the study

The aim of this study is to devise an engineering procedure for calculating the radial ductility of roller bearings that would account for a no-centrally applied radial load. This would make it possible, based on relatively simple calculations, to estimate an additional increase in the ductility of bearings due to the misalignment of rings.

To accomplish the aim, the following tasks have been set:
– to improve the known procedure for calculating the radial ductility of roller bearings under a central radial load considering the contact elastic deformations of parts;
– to account for the contact deformations of parts, the radial gap, the deformations of bending, and the misalignment of rings.

4. Improvement of the procedure for calculating the radial ductility of roller bearings under a central radial load

Accepted assumptions:
– rollers and rings are manufactured from a homogeneous isotropic material and are of perfect cylindrical shape;
– there are no roughness and undulation in the parts' surfaces;

– there are no bending deformations in rings; only the contact deformations of rings and rollers are considered;

– the influence of lubrication is not taken into consideration.

The elastic contact deformation of two cylindrical bodies at compression by force F_0 is determined from known expression [11]:

$$\delta_{r0} = C_{r0} \cdot F_0, \qquad (1)$$

where C_{r0} is the rigidity coefficient,

$$C_{r0} = \frac{0.579}{l \cdot E}\left[\ln \frac{1.727 \cdot l \cdot E \cdot (R_1 + R_2)}{F_0} + 0.814\right],$$

where E is the modulus of elasticity; R_1, R_2 are the cylinders' radii, respectively; l is the length of the cylinders.

The radial ductility δ_{r0} (elastic contact deformation) of cylindrical roller bearings under radial load F_r is determined from the sum of elastic deformations δ_{r0i} and δ_{r0o} in contacts between the central most loaded roller and the inner and outer rings, compressed by force F_0:

$$\delta_{r0} = \delta_{r0i} + \delta_{r0o}, \qquad (2)$$

where $\delta_{r0i} = C_{r0i} \cdot F_0$, $\delta_{r0o} = C_{r0o} \cdot F_0$.

Magnitudes: C_{r0i}, C_{r0o} are the rigidity coefficients of the contact between a roller and the inner and outer rings, respectively:

$$C_{r0i} = \frac{0.579}{l_r \cdot E}\left[\ln \frac{1.727 l_r \cdot E \cdot (R_i + R_r)}{F_0} + 0.814\right],$$

$$C_{r0o} = \frac{0.579}{l_r \cdot E}\left[\ln \frac{1.727 l_r \cdot E \cdot (R_o + R_r)}{F_0} + 0.814\right].$$

Magnitudes: R_o is the radius of the outer ring's running track; R_i is the radius of the inner ring's running track; R_r is the radius of a roller; l_r is the length of the roller's working section.

The refined magnitude of the force acting on a central roller is determined by taking into consideration a mismatch between the geometric centers of the outer and inner rings in accordance with the formula reported in paper [12]:

$$F_0 = \frac{F_r}{\left(1 + 2\sum_{j=1}^{n} C_{r0} \cdot \cos\gamma'_j\right)}, \qquad (3)$$

where γ_j is the contact angle between the j-th roller and an inner ring; n is the number of rollers in the loaded zone of a bearing.

5. Devising an engineering procedure for calculating the radial ductility of roller bearings under a no-centrally applied radial load

Accepted assumptions:

– rollers and rings are manufactured from a homogeneous isotropic material and are of perfect cylindrical shape;

– there are no roughness and undulation in the parts' surfaces;

– there is no bending deformation of the inner ring, which is more common during operation (the inner ring is coupled to the shaft of solid section via a tight mounting); only the contact deformations of rings and rollers are considered;

– the misalignment of rings forms under a no-centrally applied radial load;

– the small angles in the misalignment of rings are considered, when a contact between a roller and the rings is maintained throughout its length;

– the linear law of change in the deformations in contacts between a central roller and the rings holds for small angles in the misalignment of rings;

– the influence of lubrication is not taken into consideration.

The radial ductility of cylindrical roller bearings is similarly determined from expression (2). The magnitude for force F_0 in this case is determined by taking into consideration the radial gap G_r, the bending deformation u_o of the outer ring and takes the following form [12]:

$$F_0 = F_r \left/ \left(1 + 2\sum_{j=1}^{n} C_{r0} \cdot \left(\cos\gamma'_j + \frac{G_r}{\delta_{r0}}\left(\cos\gamma'_j - 1\right) - \frac{1}{\delta_{r0}}u_o\right)\right)\right. \qquad (4)$$

The misalignment of bearing rings caused by a no-centrally applied radial load F_r is accounted for by applying an estimation model shown in Fig. 1.
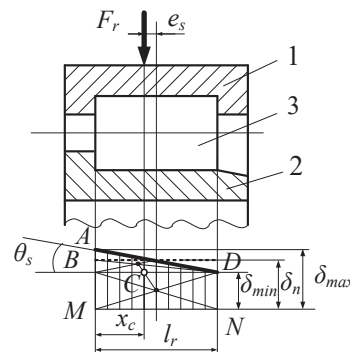

Fig. 1. Estimation model for determining the angle of rings misalignment under a no-centrally applied radial load on a bearing: 1 — outer ring; 2 — inner ring; 3 — cylindrical roller

The emergence of eccentricity e_s in the load application F_r leads to that the load distribution along the generatrix of the roller is uneven and there occurs the misalignment of rings defined by angle θ_s. By denoting the area of rectangle $BDNM$ through S_1, the area of triangle ABD – S_2, a position of the center of gravity x_c of trapezoid $ADNM$ is determined from:

$$x_c = \frac{S_1 \cdot \frac{1}{2} \cdot l_r + S_2 \cdot \frac{2}{3} \cdot l_r}{S_1 + S_2}. \qquad (5)$$

Taking into consideration magnitudes $S_1 = \delta_{min} \cdot l_r$ and $S_2 = \delta_{max} \cdot l_r$, expression (5) takes the form:

$$x_c = \frac{l_r \cdot (\delta_{min} + 2\delta_{max})}{3(\delta_{min} + \delta_{max})}. \qquad (6)$$

Values δ_{min} and δ_{max} for a change diagram of contact deformations take the form:

$$\delta_{min} = \delta_n - 0.5\theta_s \cdot l_r, \qquad (7)$$

$$\delta_{max} = \delta_n + 0.5\theta_s \cdot l_r, \qquad (8)$$

where $\delta_n = 8.0 \cdot 10^{-5} \dfrac{F_r^{0.9}}{l_r^{0.8}}$ is the rated contact deformation of a roller [13].

Considering (7), expression x_c is determined from:

$$x_c = \frac{l_r}{6}\left(3 + \frac{\theta_s \cdot l_r}{2\delta_n}\right). \qquad (9)$$

Considering $x_c = e_s + 0.5 l_r$ (Fig. 1), the magnitude θ_s takes the resulting form:

$$\theta_s = \frac{12\delta_n \cdot e_s}{l_r^2}. \qquad (10)$$

To find the maximum magnitude for the radial ductility of cylindrical roller bearings, the value for δ_{r0}, derived using expressions (1), (2), (4), replaces δ_n in formulae (8) and (10).

6. Results of research into devising an engineering procedure to calculate the ductility of roller bearings

Fig. 2 shows the results from calculating specific ductility δ' ($\delta' = \delta_{r0}/q$) of the cylindrical roller bearing 2211 with a central specific radial load q, based on data from paper [4] and based on the proposed procedure.
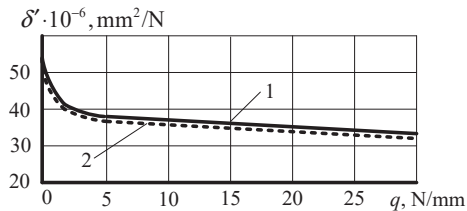
Fig. 2. Dependences of specific ductility of the bearing 2211 on specific load: 1 — according to [4]; 2 — based on the proposed procedure

A comparison of the known results from calculating specific ductility of the bearing 2211 [4] with the results from calculations based on the proposed procedure reveals that the values from the former are 3...4 % higher than those from the latter.

Our study into the influence of a no-centrally applied radial load on radial ductility was performed for the cylindrical roller bearing 42726 with the following geometrical parameters: $R_r = 16$ mm, $l_r = 42$ mm, $R_i = 79$ mm, $R_o = 111$ mm, and the following loading conditions: $F_r = 50$ kN; $e_s = 0$; $e_s = l_r/10$; $e_s = l_r/8$. The effect of the number of rollers z, the magnitude of radial gap G_r and the rigidity (the moment of inertia of an outer ring) I of the outer ring on radial ductility δ_{r0} of a bearing at $e_s = 0$, $e_s = l_r/10$ and $e_s = l_r/8$, is shown in Fig. 3.

An analysis of dependences in Fig. 3 shows a decrease in the bearing ductility with an increasing number of rollers and rigidity of the outer ring. The eccentricity of the radial load significantly (by 38...43 %) enhances the bearing ductility.
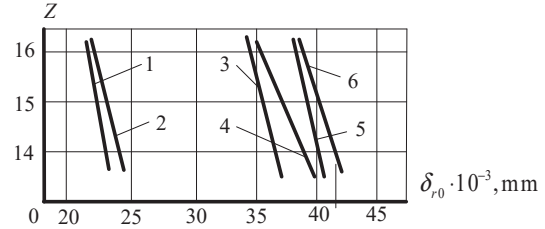
Fig. 3. Dependences of ductility of the bearing 42726 on the number of rollers and the inertia moment of the outer ring at load eccentricity: 1 — $e_s = 0$; $I = 209.6$ cm^4; 2 — $e_s = 0$; $I = 26.2$ cm^4; 3 — $e_s = l_r/10$ cm; $I = 209.6$ cm^4; 4 — $e_s = l_r/10$ cm; $I = 26.2$ cm^4; 5 — $e_s = l_r/8$ cm; $I = 209.6$ cm^4; 6 — $e_s = l_r/8$ cm; $I = 26.2$ cm^4

7. Discussion of research results of devising an engineering procedure to calculate the ductility of roller bearings

The proposed engineering procedure for calculating the radial ductility of roller bearings is implemented by solving two problems.

In the first problem, we have improved known calculation procedure for radial ductility of roller bearings under a central radial load by accounting for the mismatch between the centers of the inner and outer rings. That has made it possible to refine the magnitude of a radial force acting on the most loaded roller according to paper [12]. Adequacy of the proposed engineering procedure has been confirmed by results from calculating specific ductility of the cylindrical roller bearing 2211 under a central radial load. Using the proposed procedure, we have derived values for specific ductility that are 3...4 % lower compared to similar results when applying a known procedure.

In the second problem, we have devised an engineering procedure for calculating the radial ductility of roller bearings under a no-centrally applied radial load, considering the contact deformations of parts, the radial gap, as well as the bending deformations of rings. This has allowed us to analytically determine, based on engineering calculations, the bearing ductility under a no-centrally applied radial force that causes the misalignment of rings. Using the cylindrical roller bearing 42726 as an example, we studied design parameters considering a no-central radial load. That revealed a decrease in the bearing 42726 ductility with an increase in the number of rollers and rigidity of the outer ring, as well as with a decrease in the eccentricity of a radial load.

The current study's limitation is that it does not take into consideration the actual structure of a parts' material, the roughness and undulation in the parts' surfaces, as well as lubrication. In addition, the work addressed the simplest case in the formation of misalignment of rings in a bearing, arising from a no-centrally applied radial load.

The disadvantage of the reported study is the simplified analytical model to determine the angle of misalignment of rings under a no-centrally applied radial load on the bearing. This model could be used only for cases of small angles of misalignment, when a contact between a roller and the rings is maintained throughout its length. It is considered that the linear law of change in deformations in contacts between a central roller and the rings holds in this case. Indeed, even at small angles of rings misalignment the distribution of elastic contact deformations is non-linear. The larger angles

of misalignment in rings, when one of the edges of a roller is released from the radial load, require special research into the contact problem.

The current study could be further advanced to refine the magnitudes for contact deformations for large and small angles of rings misalignment under simple, as well as more complex, schemes of loading a bearing. The cases of jointly applied radial and axial central or no-central forces require solving a problem on the distribution of axial force along the rollers. In this case, depending on the magnitude and position of the eccentricity of the applied axial force, the problem on calculating the ductility of a bearing could become significantly complicated. In a general case, there is a need to construct a method for calculating the combined (radial and axial) ductility of cylindrical roller bearings caused by the misalignment of rollers both in a normal plane to the rolling track of an inner ring and in the tangent plane [14].

The presented engineering procedure for calculating the radial ductility of roller bearings accounting for the patterns in their operation could be applied in the practice of designing machines and mechanisms for which the elastic characteristics of all their elements are important.

8. Conclusions

1. The improvement of a known procedure for calculating the radial ductility of roller bearings implies accounting for the mismatch between the centers of the inner and outer rings. That has made it possible to refine the magnitude for a radial force at the most loaded roller. The calculation of specific ductility for the bearing 2211 under a central radial load based on known procedure has revealed that the values are 3...4 % larger in comparison with the improved methodology.

2. The radial ductility of cylindrical roller bearings with the eccentricity in a radial load is defined by the sum of variable elastic deformations of the rings and the most loaded roller in line with known formulae for the contact problem. The accepted linear estimation model for determining the angle of rings misalignment is valid for small magnitudes of angles, when a contact is maintained along the entire length of the roller. Using the bearing 42726 as an example, we have shown a decrease in the bearing ductility with an increase in the number of rolling bodies and rigidity of the outer ring, as well as with a decreasing eccentricity in the radial load.

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